A Novel Alignment Accuracy Evaluation Approach for Shipborne Slave INS

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***Abstract*—Transfer alignment is a moving base alignment method which has been widely used in shipborne low-accuracy inertial navigation system (INS). The accuracy of transfer alignment has mainly determined the entire navigation performance of low-accuracy INS and should be obtained. In this paper, transfer alignment accuracy evaluation methods for shipborne salve INS are analyzed. Observability analysis of evaluation system is implemented, and its validity on the illustration and prediction for the potential smoothing results are verified. The relationships between accelerated motion, rotation observation variable and the performance of evaluation approaches are obtained. Then, a novel transfer alignment accuracy evaluation approach is presented. The azimuth information provided by differential GPS (DGPS) is introduced together with the velocity and position provided by DPGS as the observation variables. Kalman filter and RTS smoother are applied to process these data. Simulations are conducted and the results show that this method exhibits excellent performance, and all the errors of smoothed estimations are less than 1 percent, especially with a better estimation result of the azimuth misalignment angle.**

Keywords*—INS; transfer alignment; Kalman filter; RTS smoother

I. INTRODUCTION

Transfer alignment is a moving base alignment method which is suitable for shipborne low-accuracy inertial navigation system (INS). However, the accuracy of transfer alignment is restrained by many factors, such as the error of shipborne master INS, the effect of level-arm and the dynamic flexure of ship body. Therefore, the technique of alignment accuracy evaluation is required to obtain the performance of each alignment scheme, which would benefit the design and betterment of transfer alignment procedures [1]. And the aim of alignment accuracy evaluation is to estimate the misalignment angles after the alignment process.

Similar with the methods of alignment for shipborne slave INS, most of the existing accuracy evaluation approaches in the ship field are developed from the ones used in aviation [2,3]. In these methods, the residual misalignment angles are estimated by utilizing Kalman smoothing algorithms. The velocity and position provided by DGPS are employed as reference measurements for evaluation. However, compared with airplanes, the ships suffer from non-ignorable resistance caused by water, which makes the ship exhibited weak maneuverability. If this kind of approach is applied in ships, the attitude misalignment angles could not be precisely smoothed. Especially, the azimuth misalignment angle cannot be effectively estimated due to the limited maneuverability of ships [4].

To solve this problem, an available solution is to extend the observation variables with rotation information. In [1], the attitude information provided by master INS is introduced as the additional matching reference information. However, the attitude difference between master INS and slave INS would be gravely impacted by the dynamic flexure of ship. Wide range flexural amplitude would directly lead to the invalidity of this method.

In this paper, a novel transfer alignment accuracy evaluation method is proposed. Section 2 introduces the state estimation methods involved in the typical accuracy evaluation approaches. In section 3, based on statement of the evaluation system model, observability analysis of this system is conducted, and its validity on the illustration and prediction for the potential smoothing results are verified. Then, a novel evaluation method by introducing azimuth information provided by DGPS is presented in section 4. In section 5, the performance of this new method is given out by both observability analysis and simulations. Finally, the conclusion and perspective future work are outlined in section 6.

II. THE STATE ESTIMATION METHODS INVOLVED IN TRANSFER ALIGNMENT ACCURACY EVALUATION

In the optimal estimation theory, smoothing means using the observation data we obtained before to estimate the states for each time steps. In the evaluation problem, when the

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alignment process finished, the alignment errors including the misalignment angles would become the initial errors for the navigation of slave INS and propagate with the common navigation error characteristics. Therefore, it is available to manipulate the evaluation problem by observing the output errors of slave INS to smooth the alignment errors. Kalman fixed point smoother and RTS fixed interval smoother are frequently used in evaluation system. In this paper, RTS smoothing algorithm is selected and the evaluation process is conducted offline.

A. *Classical Kalman Filter*

Assuming that the state equations of discrete system are:

$$\begin{cases} \boldsymbol{X}_k = \boldsymbol{\Phi}_{k,k-1}\boldsymbol{X}_{k-1} + \boldsymbol{W}_{k-1}, \\ \boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{V}_k. \end{cases} \tag{1}$$

Where $\boldsymbol{X}_k$ is state vector, $\Phi_{k,k-1}$ is state transition matrix, $\boldsymbol{W}_{k-1}$ is zero-mean system noise, $\boldsymbol{Z}_k$ is measurement vector, $\boldsymbol{H}_k$ is observation operator, $\boldsymbol{V}_k$ is zero-mean measurement noise.

Based on the Gaussian linear assumption, Kalman filter has been developed and become one of the most classical linear filtering approaches [5], which can be shown as following steps:

(1) Prediction

Calculating one-step prediction $\hat{\boldsymbol{X}}^f_{k,k-1}$ and its covariance $\boldsymbol{P}^f_{k,k-1}$:

$$\begin{cases} \hat{\boldsymbol{X}}^f_{k,k-1} = \boldsymbol{\Phi}^f_{k,k-1}\hat{\boldsymbol{X}}^f_{k-1} \\ \boldsymbol{P}^f_{k,k-1} = \boldsymbol{\Phi}^f_{k,k-1}\boldsymbol{P}^f_{k-1}[\boldsymbol{\Phi}^f_{k,k-1}]^T + \boldsymbol{Q}_{k-1} \end{cases} \tag{2}$$

(2) Update

Calculating the gain matrix of Kalman filter $\boldsymbol{K}^f_k$, and updating the estimate of states $\hat{\boldsymbol{X}}^f_k$ and its covariance $\boldsymbol{P}^f_k$ by using measurements:

$$\begin{cases} \boldsymbol{K}^f_k = \boldsymbol{P}^f_{k,k-1}\boldsymbol{H}^T_k[\boldsymbol{H}_k\boldsymbol{P}^f_{k,k-1}\boldsymbol{H}^T_k + \boldsymbol{R}_k]^{-1} \\ \hat{\boldsymbol{X}}^f_k = \hat{\boldsymbol{X}}^f_{k,k-1} + \boldsymbol{K}^f_k[\boldsymbol{Z}_k - \boldsymbol{H}_k\hat{\boldsymbol{X}}^f_{k,k-1}] \\ \boldsymbol{P}^f_k = [\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k]\boldsymbol{P}^f_{k,k-1}[\boldsymbol{I} - \boldsymbol{K}^f_k\boldsymbol{H}_k]^T + \boldsymbol{K}^f_k\boldsymbol{R}_k[\boldsymbol{K}^f_k]^T \end{cases} \tag{3}$$

Where superscript f denotes the filtering process, and $k=1,2,\cdots,N$.

B. *RTS Fixed Interval Smoother*

Based on the classical Kalman filter, a classical interval smoother was proposed by Rauch, Tung and Striebelrts in 1965, which is also called RTS fixed interval smoother [6].

In the Bayesian theory, the fixed interval smoothing problem can be expressed by using conditional probability distribution as:

$$p(\boldsymbol{X}_k \mid \boldsymbol{Z}_{1:N}) \tag{4}$$

Based on the conditional probability density formula, the joint probability density of arbitrate two adjacent moments can be expressed as:

$$\begin{aligned} p(\boldsymbol{X}_k, \boldsymbol{X}_{k+1}, \boldsymbol{Z}_{1:N}) &= p(\boldsymbol{X}_{k+1} \mid \boldsymbol{X}_k)\, p(\boldsymbol{X}_k \mid \boldsymbol{Z}_k) \\ &\cdot p(\boldsymbol{Z}_{k+1} ..., \boldsymbol{Z}_N \mid \boldsymbol{X}_{k+1})\, p(\boldsymbol{Z}_k) \end{aligned} \tag{5}$$

Under the gaussian assumptions, using the maximum likelihood estimation criterion, the expression of RTS fixed interval smoothing algorithm can be deduced as:

$$\begin{cases} \hat{\boldsymbol{X}}^s_k = \hat{\boldsymbol{X}}^f_k + \boldsymbol{K}^s_k[\hat{\boldsymbol{X}}^s_{k+1} - \boldsymbol{\Phi}^f_{k+1,k}\hat{\boldsymbol{X}}^f_k] \\ \boldsymbol{K}^s_k = \boldsymbol{P}^f_k[\boldsymbol{\Phi}^f_{k+1,k}]^T[\boldsymbol{P}^f_{k,k-1}]^{-1} \\ \boldsymbol{P}^s_k = \boldsymbol{P}^f_k + \boldsymbol{K}^s_k[\boldsymbol{P}^s_{k+1} - \boldsymbol{P}^f_{k+1,k}][\boldsymbol{K}^s_k]^T \\ \hat{\boldsymbol{X}}^s_N = \hat{\boldsymbol{X}}^f_N, \boldsymbol{P}^s_N = \boldsymbol{P}^f_N \end{cases} \tag{6}$$

Where k=N-1,N-2, ⋯ ⋯ ,0 , the subscript s denotes the smoothing process, $\hat{\boldsymbol{X}}^s_k$ denotes the optimal smoothed estimation at the k time step. And the aim of evaluation is to obtain the value of $\hat{\boldsymbol{X}}^s_0$.

As shown in (6), there are four matrices used in the calculation process of smoothing, including: the filtered estimation of state vector $\hat{\boldsymbol{X}}^f_k$, the state transition matrix $\boldsymbol{\Phi}^f_{k+1,k}$, estimated covariance $\boldsymbol{P}^f_k$, and one step prediction covariance $\boldsymbol{P}^f_{k+1,k}$. Accordingly, it is necessary to store them in the filtering calculation. Moreover, compared with filtering, the process of smoothing is backward. The output of smoother is a correction of the output of filter. Therefore, the availability of RTS smoother for states depends on whether these states can be estimated by Kalman filter. On the other hand, the performance of Kalman filter is usually evaluated by the observabilities of systems. And the key problems are to judge whether the system is observable or to obtain the observation degree for each state.

III. OBSERVABILITY ANALYSIS OF LINE MOTION INFORMATION MATCHING EVALUATION METHOD

A. *State model of Accuracy Evaluation System*

Assuming that the 13-dimension state variables as:

$$\begin{aligned} \boldsymbol{X} = [&\boldsymbol{\phi}_x \quad \boldsymbol{\phi}_y \quad \boldsymbol{\phi}_z \quad \delta\boldsymbol{V}_x \quad \delta\boldsymbol{V}_y \quad \delta\boldsymbol{P}_x \\ &\delta\boldsymbol{P}_y \quad \boldsymbol{\varepsilon}^b_{bx} \quad \boldsymbol{\varepsilon}^b_{by} \quad \boldsymbol{\varepsilon}^b_{bz} \quad \nabla^b_{bx} \quad \nabla^b_{by} \quad \nabla^b_{bz}]^T \end{aligned} \tag{7}$$

As for the ship filed, the 2-channel INS model is selected. And its state equation can be expressed as:

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \boldsymbol{0}_{3\times2} & \boldsymbol{C}_b^n & \boldsymbol{0}_{3\times3} \\ \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{0}_{2\times2} & \boldsymbol{0}_{2\times3} & \boldsymbol{B}_3 \\ \boldsymbol{0}_{2\times3} & \boldsymbol{I}_{2\times2} & \boldsymbol{0}_{2\times2} & \boldsymbol{0}_{2\times3} & \boldsymbol{0}_{2\times3} \\ \boldsymbol{0}_{6\times3} & \boldsymbol{0}_{6\times2} & \boldsymbol{0}_{6\times2} & \boldsymbol{0}_{6\times3} & \boldsymbol{0}_{6\times3} \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} -\boldsymbol{C}_b^n \boldsymbol{\varepsilon}_w^b \\ \boldsymbol{C}_b^n \nabla_w^b \\ \boldsymbol{0}_{6\times1} \end{bmatrix} \quad (8)$$

Where

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & \boldsymbol{\omega}_{in}^z & -\boldsymbol{\omega}_{in}^y \\ -\boldsymbol{\omega}_{in}^z & 0 & -\boldsymbol{\omega}_{in}^x \\ \boldsymbol{\omega}_{in}^y & \boldsymbol{\omega}_{in}^x & 0 \end{bmatrix}, \boldsymbol{A}_2 = \begin{bmatrix} 0 & -1/R_M \\ 1/R_N & 0 \\ \tan\varphi / R_N & 0 \end{bmatrix}$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 & -\boldsymbol{f}_z^n & \boldsymbol{f}_y^n \\ \boldsymbol{f}_z^n & 0 & -\boldsymbol{f}_x^n \end{bmatrix},$$

$$\boldsymbol{B}_2 = \begin{bmatrix} 0 & \boldsymbol{\omega}_{in}^z + \boldsymbol{\omega}_{en}^z \\ -(\boldsymbol{\omega}_{in}^z + \boldsymbol{\omega}_{en}^z) & 0 \end{bmatrix},$$

$$\boldsymbol{B}_3 = \begin{bmatrix} \boldsymbol{C}_b^n(1,1) & \boldsymbol{C}_b^n(1,2) & \boldsymbol{C}_b^n(1,3) \\ \boldsymbol{C}_b^n(2,1) & \boldsymbol{C}_b^n(2,2) & \boldsymbol{C}_b^n(2,3) \end{bmatrix}.$$

In traditional alignment accuracy evaluation methods, the velocity and position provided by DGPS are commonly selected as reference information, and the measurement equation can be expressed as:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{V}_x^{INS} - \boldsymbol{V}_x^{DGPS} \\ \boldsymbol{V}_y^{INS} - \boldsymbol{V}_y^{DGPS} \\ \boldsymbol{P}_x^{INS} - \boldsymbol{P}_x^{DGPS} \\ \boldsymbol{P}_y^{INS} - \boldsymbol{P}_y^{DGPS} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0}_{2\times3} & \boldsymbol{I}_{2\times2} & \boldsymbol{0}_{2\times8} \\ \boldsymbol{0}_{2\times5} & \boldsymbol{I}_{2\times2} & \boldsymbol{0}_{2\times6} \end{bmatrix} \boldsymbol{X} + \boldsymbol{V} \quad (9)$$

Where the subscripts of *INS* and *DGPS* denote the slave INS and DGPS, respectively.

B. Observability Analysis of Accuracy Evaluation System

Singular value decomposition (SVD) based observability analysis method can effectively distinguish the observable states from the unobservable states by giving out the observation degrees of all states. Therefore, in researches for the application of Kalman filter in the inertial navigation filed, observability analysis method by combining the SVD and piece-wise constant system (PWCS) theory has been widely used [7]. When applying this method in the evaluation system, two main problems are involved: (1) the influence of matching schemes on the observability; (2) the influence of maneuvers on the observability.

Firstly, to resolve the problem that evaluation performance are restricted by the limited maneuverability of ships, the attitude information of master INS are adopted in majority of existing evaluation methods. On the condition of no maneuver, different matching schemes can be analyzed by the data provided in their corresponding single motion pieces. It is also available that two or more motion pieces are selected. However, simulation results show that there are little changes on the observation degree for each state.

Secondly, when two motion phases are involved in each evaluation method, PWCS theory would be needed to separate the evaluation process according to different motion conditions. There are two common motion manners in evaluation schemes: uniform motion and accelerated motion. Therefore, it is rational to divide the system into two pieces.

Considering of this fact, observability analysis are conducted from the systems in which maneuvers or attitudes are introduced.

The evaluation approaches are given in the table 1. And the observability analysis results are shown in table 2. As for the horizonal attitude errors, the observation degrees of $\boldsymbol{\phi}_x$ and $\boldsymbol{\phi}_y$ are almost the same. Therefore, only that of $\boldsymbol{\phi}_x$ are given out .

TABLE I. DESCRIPTION OF SCHEMES FOR OBSERVABILITY ANALYSIS

Scheme	Item	Description
(1)	Reference information	“velocity + position” from DGPS
	Motion manner	0-10s uniform motion along with 45° heading angle, with constant velocity of 4 Knot
(2)	Reference information	“velocity + position” from DGPS
	Motion manner	0-10s uniform motion along with 45° heading angle, with velocity of 4 Knot; 10-15s accelerated motion along with 45° heading angle, with acceleration of 0.3m/s^2; then, keeping uniform motion
(3)	Reference information	“velocity + position” from DGPS, horizontal attitudes from master INS
	Motion manner	anchoring
(4)	Reference information	“velocity + position” from DGPS, “horizontal + azimuth attitudes” from master INS
	Motion manner	anchoring

TABLE II. THE RESULTS OF SYSTEM OBSERVABILITY ANALYSIS

Scheme	Piece	Singular Value of $\boldsymbol{\phi}_x$	Singular Value of $\boldsymbol{\phi}_z$
(1)	Piece 1	13.9426452484	0.0000000000
	Piece 2	19.7179912310	0.0000193405
(2)	Piece 1	13.9422859936	0.0000000000
	Piece 2	19.7175119193	0.3152337755
(3)	Piece 1	13.9782551592	0.0000000000
	Piece 2	19.7680142465	0.0004243193
(4)	Piece 1	13.9785542525	0.9999999986
	Piece 2	19.7681995078	1.4142135835

According to the table 2, for all four schemes, the singular value of ϕ_x are greater than 13, which means ϕ_x can be totally observed and precisely smoothed. However, for the singular value of ϕ_z in the piece 1, there is only the value of scheme (4) appears to be approximate 1, with others are almost zeros. Therefore, there is only scheme (4) can smooth ϕ_z in piece 1, which means the smoothed estimate curve of ϕ_z can quickly converge. On the other side, compared with the values in piece 1, the singular values for ϕ_x and ϕ_z in all schemes are increased. However, in piece 2, the singular values of schemes (1) and (3) are still far less than zero. As a result, without maneuvers and additional attitude observe variables, ϕ_z can not be efficiently observed and estimated.

In the scheme (2), that's because of the accelerated maneuver, the singular value of ϕ_z significantly increases to nearly 0.3. However, due to the smaller absolute acceleration, the estimation precision is still non-ideal. In the scheme (4), the singular value of ϕ_z grows up to be greater than 1 in its piece 2, which means once if the reference attitude are accurate and stable the azimuth misalignment angle ϕ_z can be precisely smoothed for evaluation.

In order to prove the validity of observation analysis on the illustration and prediction for the potential smoothing results, simulations for the above four schemes are conducted. Taking the estimation of ϕ_z as an example, the filtering results and smoothing results are shown in Figure 1 and Figure 2, respectively.

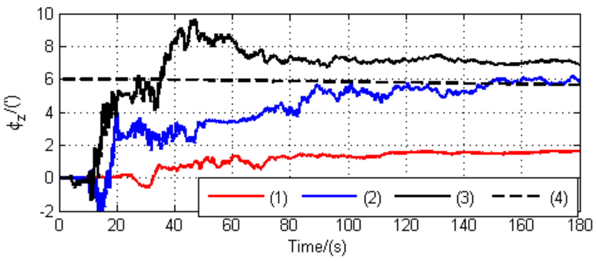

Figure 1. The filtering results of ϕ_z

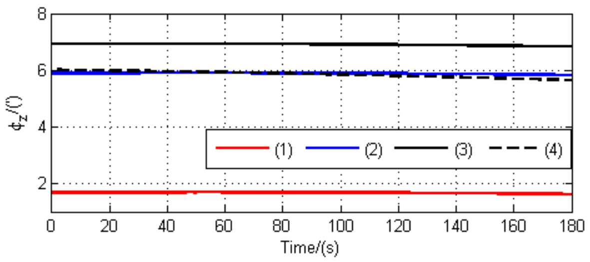

Figure 2. The smoothing results of ϕ_z

From Figure 1, except for that of scheme (4), the filtering curves of ϕ_z for the other three schemes do not happen to be rapid and precise convergence. Consequently, their smoothed estimations exhibit obvious errors in Figure 2. However, it is noted that the smoothing result of ϕ_z in scheme (3) appears to be accurate. Due to its filtering curves have not been convergent, it is occasional and meaningless.

After a compare between the simulated curves and the observability analysis results, we draw the primary conclusions as follows:

- Observability analysis is an effective mathematic tool which can bridge the relationship between evaluation schemes and smoothing performances without entire filtering and smoothing calculations.
- If the same motion of ship has been confirmed, the reference measurement matching methods by combining linear motion information with rotation information have a better smoothing performance than that only linear motion information are adopted.
- If the same measurements are taken, accelerated maneuver manners are helpful for the effective evaluation of azimuth misalignment angle.

As a result, introducing either accelerated motion or additional matching reference attitude could effectively improve the performance of accuracy evaluation approaches, especially for the azimuth misalignment angle.

IV. Novel Evaluation Method by Introducing Azimuth Misalignment Angle

Aim to solve the smoothed estimation problem for the azimuth misalignment angle, a novel alignment accuracy evaluation approach by adding the reference azimuth provided by DGPS is presented. There are many types of equipment on ship which can provide accurate attitude information, such as shipborne master INS and DGPS. If the attitudes offered by shipborne master INS are the formal adoption, the real-time dynamic flexure of ship body has to be compensated online. However, it is very difficult to be measured or estimated due to its untraceable stochastic characteristics on ship's different motion conditions.

DGPS appears a better attitude source, which provides reference velocity and position in traditional evaluation methods. It is convenient to realize a precise measurement for the heading attitude of ship by using two GPS receiving antennas. The problem of dynamic flexure compensation can be evaded, once the antennas and shipborne slave INS are mounted on the same foundation base as closely as possible. So it is an ideal reference information source for the evaluation system. In this new evaluation approach, the combinational reference are constructed by the DGPS which provide with the position and velocity information of ship and the azimuth angle of ship.

Selecting the position difference and velocity difference together with the azimuth difference between slave INS and DGPS as the observation variables, the measurement equation can be expressed as:

$$Z' = \begin{bmatrix} V_x^{INS} - V_x^{DGPS} \\ V_y^{INS} - V_y^{DGPS} \\ P_x^{INS} - P_x^{DGPS} \\ P_y^{INS} - P_y^{DGPS} \\ H^{INS} - H^{DGPS} \end{bmatrix} = \begin{bmatrix} 0_{2\times3} & I_{2\times2} & 0_{2\times8} \\ 0_{2\times5} & I_{2\times2} & 0_{2\times6} \\ 0_{1\times2} & 1 & 0_{1\times8} \end{bmatrix} X + V \quad (10)$$

Where H denotes the azimuth angle.

On the other side, to strengthen the carrier accelerated motion can also be used to further improve the estimation precision of ϕ_z. Analyzing from the system equations, in order to improve attitude accuracy evaluation performance of all misalignment angles, to enlarge the matrix elements in A_1, B_1 which are approximately zero value appears to be an effective solution. Through this method, the scales of attitude errors in system equations are enlarged as well. Finally, the contribution of system equations on the estimated results is improved rather than being directly influenced by measurements.

V. SIMULATION

A. Simulation Conditions

Assuming the gyroscope constant drifts are $0.1\ ^\circ/\mathrm{h}$, the initial residual misalignment angles are $6'$. Simulations are conducted as the three following schemes in Table 3.

TABLE III. SCHEMES ILLUSTRATION FOR SIMULATIONS

Scheme	Item	Description
(a)	Reference information	“velocity + position” from DGPS
	Motion manner	0-10s uniform motion along with 45° heading angle, with velocity of 4 Knot; 10-15s accelerated motion along with 45° heading angle, with acceleration of $0.3\mathrm{m/s^2}$; then, keeping uniform motion
(b)	Reference information	“velocity + position + azimuth angle” from DGPS
	Motion manner	anchoring
(c)	Reference information	“velocity + position + azimuth angle” from DGPS
	Motion manner	The same with Scheme (a)

The filter matrices are setted as following：

$$P(0) = \mathrm{diag}\{(6')^2, (6')^2, (6')^2, (0.01\ \mathrm{m/s})^2, (0.01\ \mathrm{m/s})^2, (5\ \mathrm{m})^2, (8\ \mathrm{m})^2, (0.1^\circ/\mathrm{h})^2, (0.1^\circ/\mathrm{h})^2, (0.1^\circ/\mathrm{h})^2, (100\ \mu\mathrm{g})^2, (100\ \mu\mathrm{g})^2, (100\ \mu\mathrm{g})^2\} \quad (11)$$

$$Q = \mathrm{diag}\{(0.05^\circ/\mathrm{h})^2, (0.05^\circ/\mathrm{h})^2, (0.05^\circ/\mathrm{h})^2, (50\ \mu\mathrm{g})^2, (50\ \mu\mathrm{g})^2, (50\ \mu\mathrm{g})^2, 0,0,0,0,0,0,0\} \quad (12)$$

The noise matrix in conventional scheme (a) is

$$R = \mathrm{diag}\{(0.01\ \mathrm{m/s})^2, (0.01\ \mathrm{m/s})^2, (6\ \mathrm{m})^2, (6\ \mathrm{m})^2\} \quad (13)$$

The new noise matrix in the novel scheme by introducing the azimuth information offered by DGPS can be expressed as:

$$R' = \mathrm{diag}\{(0.01\ \mathrm{m/s})^2, (0.01\ \mathrm{m/s})^2, (6\ \mathrm{m})^2, (6\ \mathrm{m})^2, (0.01^\circ)^2\} \quad (14)$$

B. Results

Observability analysis are implemented and the results are shown in Table 4. Compared with that of the schemes (a) and (b), the singular value of ϕ_z in scheme (c) are the greatest in piece 2.

TABLE IV. THE RESULTS OF SYSTEM OBSERVABILITY ANALYSIS

Scheme	Piece	Singular Value of ϕ_x	Singular Value of ϕ_z
(a)	Piece 1	13.9422859936	0.0000000000
	Piece 2	19.7175119193	0.3152337755
(b)	Piece 1	13.9421100277	0.9999970670
	Piece 2	19.7171735017	1.4142095911
(c)	Piece 1	13.9428595625	0.9999971903
	Piece 2	19.7179632128	1.4457253370

Simulation results are given out by Table 5, Figure 3 and Figure 4.

TABLE V. THE RESULTS OF ALIGNMENT ACCURACY EVALUATION

Scheme	ϕ_x		ϕ_z	
	Smoothed value	***Error***	***Smoothed value***	***Error***
(a)	5.968′	0.53%	5.206′	13.23%
(b)	5.996′	0.07%	6.004′	0.07%
(c)	5.966′	0.57%	6.007′	0.12%

Due to the misalignment angles at the end of transfer alignment process are manipulated as the initial attitude errors in the alignment accuracy evaluation process, it is the very value we desired corresponding to the smoothed values at the zero time step.

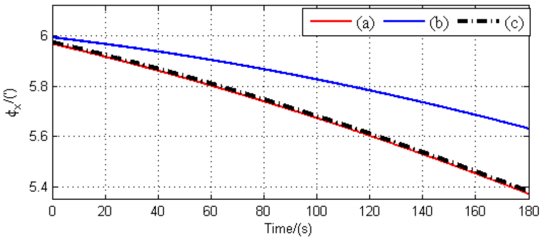

Figure 3. The smoothed value of ϕ_x

According to Table 4, Table 5 and Figure 3, all the three schemes can precisely smoothed estimate ϕ_x approximately to the initial residual misalignment angle 6', with estimation errors less than 1 precent.

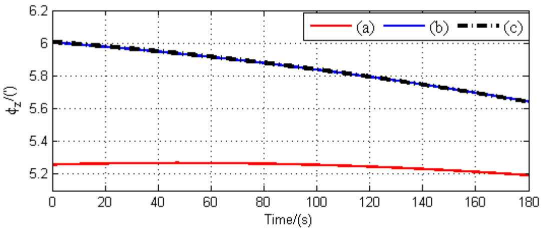

Figure 4. The smoothed value of ϕ_z

In Figure 4, for the conventional scheme (a) , the precise of smoothed estimate for ϕ_z is weak. What's more serious, the corresponding filtering curve appears not convergent in simulation. Therefore, the smoothed estimate value of ϕ_z by using scheme (a) exhibits a bad stability of data as well as too lager smoothed error. The main reason of that is the ship's limited maneuverability leading to a bad observability of ϕ_z, which can meet the requirement for precise evaluation. By introducing the azimuth information, scheme (b) and (c) appear to be with better evaluation performance, especially for the azimuth residual misalignment angle ϕ_z changing from can not effectively estimated to precisely estimated, and its error less than 1 precent.

In conclusion, the alignment accuracy evaluation approach shows excellent evaluation performance. Smoothed estimation errors for all attitude angles are less than 1 precent. And the results behave quite good stability. Moreover, as a better evaluation performance of the azimuth angle is required, the scheme of introducing both azimuth information and accelerated motion can be considered.

VI. CONCLUSION

The conclusions are listed as follows:

By analyzing the optimal estimate algorithms, the validity of observability analysis on the illustration and prediction for the potential smoothing results are verified.

Observability analysis for evaluation schemes are conducted, it is shown that by introducing either accelerated motion or additional matching reference attitude could effectively improve the singular values of the residual misalignment angles, to improve the performance of evaluation.

Simulation results show that the new alignment accuracy evaluation approach could effectively estimate the residual misalignment angles, and estimation errors in evaluation are less than 1 precent. Especially, the estimation performance of the azimuth angle is greatly improved.

The future work aims to develop the forward smoothing algorithms which enable the calculation of evaluation forward and online. New methods by employing other maneuver manners of ship and high precise attitude sources would be analyzed and its experiments are expected to be implemented.

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