Research Article

A Novel method for Target Navigation and Mapping Based on Laser Ranging and MEMS/GPS Navigation

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Making the sensor rigidly mounted in the target is the common characteristic of conventional navigation system. However, it is difficult or impossible to realize for special applications. A novel new algorithm for target navigation and mapping is designed based on the position, attitude and ranging information provided by laser distance detector (LDS) and MEMS/GPS navigation, which can solve problem of the target navigation and mapping without any sensor in the target. The detailed error analysis shows attitude error of MEMS/GPS is the main error source which dominates the accuracy of the algorithm. Based on the error analysis, a calibration algorithm is designed so as to improve the accuracy to a large extent. The results show that the performance of the algorithm can realize the target positioning with the accuracy of less than 2 meters for the target within 1 kilometer.

1. Introduction

Motion state of the target can be described by its accelerometer, velocity, position and attitude. Navigation system is the most convenient device to provide these informations by means of optical navigation, electronics navigation, dynamics navigation, acoustic navigation, and so on [1]. For the mentioned navigation system, there is a common characteristic that the navigation sensor must be mounted in the target. For example, there must be optical observation device in the target which can measure the attitude vector from the target to another known point in terrestrial navigation and celestial navigation [2]. MEMS inertial navigation system must rigidly mount the gyros and accelerometers to the target so as to measure its linear and angular movement. In communication domain, the target position method based on the cell site is also realized by communication between the cell phone and the base station [3].

Sometimes, conventional navigation system cannot act the right function in special application. For the positioning of those hard to or unable to reach places, it is difficult or expensive to mount the sensor in the target. For the positing of the enemy target in military, it is impossible [4][5].

Optical measure device in mapping is a good choice for the target mapping because it is unnecessary to mount the sensor in the target. On the basis of the known self-position, the target position can be measured accurately by means of angle measurement [6]. For example, the total station can output precise slope distance from the instrument to a particular point by a theodolite integrated with an electronic distance meter (EDM). In a total station, the theodolite is used to measure angles in the horizontal and vertical planes, the electronics distance between the instrument and the point is measured by the EDM. But for the mapping and navigation of the moving targets, optical measure device is unsuitable because it can only work in a relative static state [7].

MEMS/GPS integrated navigation system has the advantages of small size, light weight and low cost, and so it can be applied in many navigation fields such as unmanned aircrafts, land vehicles and robots [8]. If the MEMS/GPS navigation system is allowed to substitute the optical measure device, the integration of the MEMS gyro can be used to describe the vector from the instrument to the target. This can make the instrument suitable for static and moving target after integration with an EDM. In this paper, a new method based on MEMS/GPS and LDS is designed to realize the mapping and navigation of static and moving target.

But due to the low accuracy of MEMS/GPS in attitude, the attitude error probably becomes the most important error source [9]. If there isn't suitable calibration method, the mapping and navigation result for the target will be severely influenced by the attitude error. Based on the detailed error analysis, calibration algorithm for the attitude error is designed so as to compensate the heading and pitch error of MEMS/GPS. The performance of the designed algorithm is evaluated by simulations and the results show that the new method exhibits excellent navigation and mapping performance.

2. Positioning Algorithm of the Target

(1) Basic definition for the coordinate systems

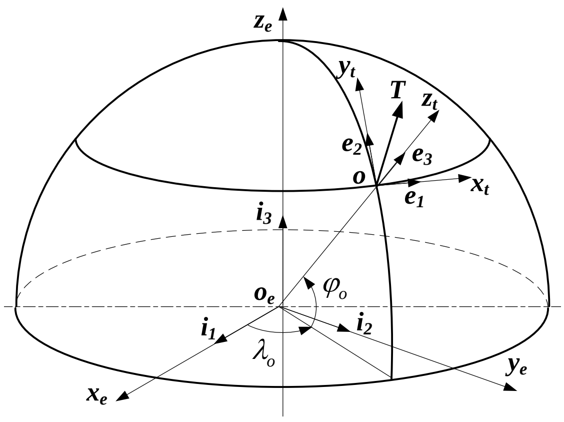


FIGURE 1: Definition for the coordinate systems

For the convenience of the observer and the target description, two coordinate systems should be defined. The first coordinate system is the earth-fixed coordinate system ($o_e x_e y_e z_e$). It is a geocentric coordinate system which is rigidly bound to the Earth (see Fig.1). Its center (o_e) is located at the geometric center of the Earth, the o_ex_e axis passes through the zero longitude in the equator plane, the o_ez_e axis is directed to the north pole and the o_ey_e axis completes the right-handed orthogonal triad system. The other coordinate system is geographic coordinate system ($ox_ty_tz_t$, see Fig.1) whose center is located in the observer, the ox_t axis is horizontal-east, the oy_t axis is horizontal-north, and the oz_t axis complete the right handed orthogonal triad system.

The relationship between $o_ex_ey_ez_e$ and $ox_ty_tz_t$ can be described by the cosine transform matrix (C_t^e) [9]:

$$C_t^e = \begin{bmatrix} \cos\lambda_o & -\sin\lambda_o & 0 \\ \sin\lambda_o & \cos\lambda_o & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_o & -\sin\varphi_o \\ 0 & \sin\varphi_o & \cos\varphi_o \end{bmatrix} \tag{1}$$

Where, φ_o, λ_o are the latitude and longitude of point *o* in $o_ex_ey_ez_e$ respectively.

(2) Basic description for the vectors

For vectors of $\boldsymbol{o_eo}$, $\boldsymbol{o_eT}$ and $\boldsymbol{oT}$ in $o_ex_ey_ez_e$, all of them can be described if the unit vector ($\boldsymbol{i}_1$, $\boldsymbol{i}_2$, $\boldsymbol{i}_3$) in $o_ex_ey_ez_e$ is involved. Thus, vectors of $\boldsymbol{o_eo}$, $\boldsymbol{o_eT}$ and $\boldsymbol{oT}$ can be described as:

$$\begin{cases} \boldsymbol{o_eo} = x_o^e\boldsymbol{i}_1 + y_o^e\boldsymbol{i}_2 + z_o^e\boldsymbol{i}_3 \\ \boldsymbol{o_eT} = x_T^e\boldsymbol{i}_1 + y_T^e\boldsymbol{i}_2 + z_T^e\boldsymbol{i}_3 \\ \boldsymbol{oT} = x_{oT}^e\boldsymbol{i}_1 + y_{oT}^e\boldsymbol{i}_2 + z_{oT}^e\boldsymbol{i}_3 \end{cases} \tag{2}$$

Where, (x_o^e, y_o^e, z_o^e) is the Cartesian coordinates of observer *o* in $o_ex_ey_ez_e$; (x_T^e, y_T^e, z_T^e) is the Cartesian coordinates of target *T* in $o_ex_ey_ez_e$; $x_{oT}^e, y_{oT}^e, z_{oT}^e$ is the coordinate difference between *T* and *o* in $o_ex_ey_ez_e$.

The three vectors in equation (2) have the following relationship:

$$\boldsymbol{o_eT} = \boldsymbol{o_eo} + \boldsymbol{oT} \tag{3}$$

From equation (3), it can be seen that the calculation of $\boldsymbol{oT}$ become the key for position calculation of the target *T* on the basis of the known position of observer *o*. This is also the first research activity.

(3) Positioning calculation for the target *T*

According to the laws of Euler angle rotation, one orthogonal coordinate system can be transformed into another orthogonal coordinate system by three rotations. For example, three rotation of H, ϕ and θ can realize the transformation from the geographic coordinate system to the body coordinate system, Where H, ϕ and θ are the angle of heading , pitch and roll respectively (see Fig.2).

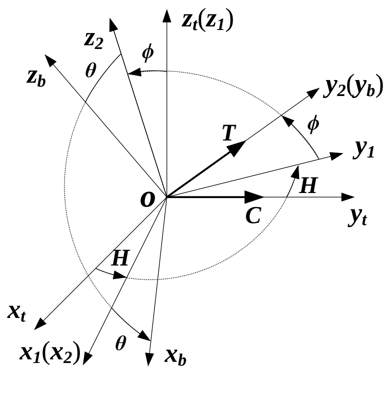


FIGURE 2: Transformation between two orthogonal coordinate systems

From Fig.2, it can be obviously found that the vector of $\boldsymbol{oT}$ is tightly concerned with the ranging between *o* and *T*, and the attitude of $\boldsymbol{oT}$ in $ox_t y_t z_t$. By involving the unit vector of $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, the vector of $\boldsymbol{oT}$ can also be described in $ox_t y_t z_t$. That is:

$$\boldsymbol{oT} = x_{oT}^t \boldsymbol{e}_1 + y_{oT}^t \boldsymbol{e}_2 + z_{oT}^t \boldsymbol{e}_3 \tag{4}$$

Where, $x_{oT}^t, y_{oT}^t, z_{oT}^t$ is the coordinate difference between *T* and *o* in $ox_t y_t z_t$.

In order to calculate the vector of $\boldsymbol{oT}$, assume a new vector of $\boldsymbol{oC}$ in $ox_t y_t z_t$.

$$\boldsymbol{oC} = 0\boldsymbol{e}_1 + d\boldsymbol{e}_2 + 0\boldsymbol{e}_3 \tag{5}$$

Where, d is the ranging between *o* and *T*.

From equation (5), the vector of $\boldsymbol{oC}$ has in fact the same direction as oy_t axis, and its length is equivalent to the ranging between *o* and *T*.

For the transformation between $\boldsymbol{oC}$ and $\boldsymbol{oT}$, two rotations of H and ϕ can accomplish the mission, because the final rotation of θ will not change the direction of $\boldsymbol{oT}$. In Fig.2, it can be seen that the oy_2 axis has the same direction as the oy_b axis.

The rotation of H and ϕ can be described by the following matrix transform. The first rotation transform matrix of H angle is:

$$\boldsymbol{C}_t^1 = \begin{bmatrix} \cos H & \sin H & 0 \\ -\sin H & \cos H & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The second rotation transform matrix of ϕ angle is

$$\boldsymbol{C}_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{7}$$

According to the definition of $\boldsymbol{oT}$ and $\boldsymbol{oC}$, the vector of $\boldsymbol{oT}$ can be expressed based on the former transform matrix.

$$\boldsymbol{oT} = \boldsymbol{C}_1^2 \boldsymbol{C}_t^1 \boldsymbol{oC} \tag{8}$$

Substituting the equation (6) and (7) into equation (8), the final calculation of $\boldsymbol{oT}$ can be got:

$$\begin{bmatrix} x_{oT}^t \\ y_{oT}^t \\ y_{oT}^t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} \cos H & \sin H & 0 \\ -\sin H & \cos H & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} = \begin{bmatrix} d \sin H \\ d \cos\phi \cos H \\ -d \sin\phi \cos H \end{bmatrix} \tag{9}$$

For the vector of ***oT***, its description in equation (2) and (4) is the same vector. On the other hand, the unit vector of $\boldsymbol{i}_1$, $\boldsymbol{i}_2$, $\boldsymbol{i}_3$ and the unit vector of $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ have the following relationship:

$$\begin{bmatrix} \boldsymbol{i}_1 \\ \boldsymbol{i}_2 \\ \boldsymbol{i}_3 \end{bmatrix} = \boldsymbol{C}_t^e \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix} \tag{10}$$

According to the relationship of equation (10), the coordinate difference between *T* and *o* in $o_e x_e y_e z_e$ can be got:

$$\begin{bmatrix} x_{oT}^e \\ y_{oT}^e \\ y_{oT}^e \end{bmatrix} = \boldsymbol{C}_t^e \begin{bmatrix} x_{oT}^t \\ y_{oT}^t \\ y_{oT}^t \end{bmatrix} \tag{11}$$

Thus, the position of the target (x_T^e, y_T^e, z_T^e) can be got based on the equation (3) and equation (11).

$$\begin{bmatrix} x_T^e \\ y_T^e \\ y_T^e \end{bmatrix} = \begin{bmatrix} x_A^e \\ y_A^e \\ y_A^e \end{bmatrix} + \boldsymbol{C}_e^t \begin{bmatrix} d \sin H \\ d \cos\phi \cos H \\ -d \sin\phi \cos H \end{bmatrix} \tag{12}$$

All the variables in equation (12) are ideal. In order to calculate the target position, the position of *o*, the distance between *o* and *T*, and the orientation of ***oT*** must be known by suitable measurement devices.

If the MEMS/GPS can be rigidly mounted in the observer along with the body coordinate system, the position of *o* can be calculated from the position output of MEMS/GPS, and the orientation of ***oT*** can be described by the attitude of MEMS/GPS. On the other hand, LDS can measure the distance if it is mounted along with the oy_b axis of the body coordinate system accurately.

Thus, define the new variables of the measurement:

1) Latitude, longitude and height measurement output of MEMS/GPS: $\tilde{\varphi}_o, \tilde{\lambda}_o, \tilde{h}_o$;

2) Heading, pitch and roll measurement output of MEMS/GPS: $\tilde{H}$, $\tilde{\phi}$, $\tilde{\theta}$;

3) Ranging measurement output of LDS: $\tilde{d}$.

After getting the measurement information of MEMS/GPS and LDS, the Cartesian coordinates of observer *o* can be calculated according with the equation (13) [10]:

$$\begin{cases} \tilde{x}_A^e = (R_N + \tilde{h}_A)\cos\tilde{\varphi}_A \cos\tilde{\lambda}_A \\ \tilde{x}_A^e = (R_N + \tilde{h}_A)\cos\tilde{\varphi}_A \sin\tilde{\lambda}_A \\ \tilde{z}_A^e = [R_N(1-e^2) + \tilde{h}_A]\sin\tilde{\varphi}_A \end{cases} \tag{13}$$

Where, $R_N = a / \sqrt{1-e^2 \sin^2 \tilde{\varphi}_o}$, a is the long axle of the Earth, e is the eccentricity of the Earth.

By substituting the ideal variables with the measurement result, the final Cartesian coordinates of the target can be got:

$$\begin{bmatrix} \tilde{x}_T^e \\ \tilde{y}_T^e \\ \tilde{z}_T^e \end{bmatrix} = \begin{bmatrix} \tilde{x}_A^e \\ \tilde{y}_A^e \\ \tilde{z}_A^e \end{bmatrix} + \begin{bmatrix} \cos\tilde{\lambda}_A & -\sin\tilde{\lambda}_A \cos\tilde{\varphi}_A & \sin\tilde{\lambda}_A \cos\tilde{\varphi}_A \\ \sin\tilde{\lambda}_A & \cos\tilde{\lambda}_A \cos\tilde{\varphi}_A & -\cos\tilde{\lambda}_A \sin\tilde{\varphi}_A \\ 0 & \sin\tilde{\varphi}_A & \cos\tilde{\varphi}_A \end{bmatrix} \cdot \begin{bmatrix} \tilde{d} \sin\tilde{H} \\ \tilde{d}\cos\tilde{\phi}\cos\tilde{H} \\ -\tilde{d}\sin\tilde{\phi}\cos\tilde{H} \end{bmatrix} \tag{14}$$

(4) Functions for the target mapping and navigation

Because MEMS/GPS and LDS can realize real-time measurement, the new algorithm can be used for both the static and the moving target. Besides the target position, the new algorithm can also measure many other informations including longitude, latitude, height of the target, the slant range from one point to point, the height difference and horizontal ranging between two targets, and so on. All these informations can all be calculated based on the position information of the target.

For the slant range ($\tilde{D}$) from one point to another point, $\tilde{D}$ can be calculated by the equation (15):

$$\tilde{D} = \sqrt{(\tilde{x}_{T2}^e - \tilde{x}_{T1}^e)^2 + (\tilde{y}_{T2}^e - \tilde{y}_{T1}^e)^2 + (\tilde{z}_{T2}^e - \tilde{z}_{T1}^e)^2} \tag{15}$$

Where, $(\tilde{x}_{T1}^e, \tilde{y}_{T1}^e, \tilde{z}_{T1}^e)$ and $(\tilde{x}_{T2}^e, \tilde{y}_{T2}^e, \tilde{z}_{T2}^e)$ are the measurement result of the Cartesian coordinates for target T_1 and T_2, respectively.

The height difference ($\Delta\tilde{H}$) between the two targets is represented as:

$$\Delta\tilde{H} = \left| \frac{\sqrt{(\tilde{x}_{T2}^e)^2 + (\tilde{y}_{T2}^e)^2}}{\cos\tilde{\varphi}_{T2}} - \frac{\sqrt{(\tilde{x}_{T1}^e)^2 + (\tilde{y}_{T1}^e)^2}}{\cos\tilde{\varphi}_{T1}} \right| \tag{16}$$

3. Error Analysis for the Algorithm

As mentioned in the former introduction, MEMS/GPS has a relatively low accuracy of attitude. For example, NAV440 GPS-aided MEMS inertial system has a 0.4 deg (rms) accuracy of pitch and roll, a 1.0 deg (rms) accuracy of heading for all motion [11]. ADIS16480 MEMS/GPS has an accuracy of 0.1 ° (pitch, roll) and 0.3 ° (heading) for static state [12]. Besides the attitude error, the positioning error of MEMS/GPS probably has a severe impact on the position calculation of the target. For example, Omni STAR HP of Trimble has accuracy of less than 10 centimeters [13]. On the other hand, LDS also has measurement error. For example, the accuracy of AccuRange AR500™ Laser Sensor is generally specified with a linearity of about +/- 0.15% of the range [14].

(1) Positioning error caused by the positioning error of MEMS/GPS

The positioning error of MEMS/GPS will caused two errors which are tightly concerned with the calculation of the target. Those are the position error of point *o* and the cosine transform matrix of $\tilde{\boldsymbol{C}}_e^t$. Based on the equation (12) and (14), the target positioning error caused by the positioning error MEMS/GPS can be calculated with the neglect of other error resources.

$$\boldsymbol{\Delta}_1 = \boldsymbol{\Delta}_{11} + \boldsymbol{\Delta}_{12} = \begin{bmatrix} \Delta x_o^e \\ \Delta y_o^e \\ \Delta z_o^e \end{bmatrix} + \Delta \boldsymbol{C}_e^t \begin{bmatrix} d\sin H \\ d\cos\phi\cos H \\ -d\sin\phi\cos H \end{bmatrix} = \begin{bmatrix} \Delta x_o^e \\ \Delta y_o^e \\ \Delta z_o^e \end{bmatrix} + \begin{bmatrix} \Delta c_{11} & \Delta c_{12} & \Delta c_{13} \\ \Delta c_{21} & \Delta c_{22} & \Delta c_{23} \\ \Delta c_{31} & \Delta c_{32} & \Delta c_{33} \end{bmatrix} \begin{bmatrix} d\sin H \\ d\cos\phi\cos H \\ -d\sin\phi\cos H \end{bmatrix} \tag{17}$$

Where, $\Delta x_o^e = \tilde{x}_o^e - x_o^e$, $\Delta y_o^e = \tilde{y}_o^e - y_o^e$, $\Delta z_o^e = \tilde{z}_o^e - z_o^e$, $\Delta \boldsymbol{C}_e^t = \tilde{\boldsymbol{C}}_e^t - \boldsymbol{C}_e^t$.

Comparing with the item of $\boldsymbol{\Delta}_{11}$, another item of $\boldsymbol{\Delta}_{12}$ can be neglected because the error of the transform matrix is very small. For example, the first line of $\Delta \boldsymbol{C}_e^t$ have the following characteristic:

$$\begin{cases} \Delta c_{11} = -\sin\lambda_o \Delta\lambda_o \le |\Delta\lambda_o| \\ \Delta c_{12} = \sin\lambda_o \sin\varphi_o \Delta\varphi_A - \cos\lambda_o \cos\varphi_o \Delta\lambda_o \le |\Delta\varphi_o| + |\Delta\lambda_o| \\ \Delta c_{13} = \sin\lambda_o \cos\varphi_o \Delta\varphi_o + \cos\lambda_o \sin\varphi_o \Delta\lambda_o \le |\Delta\varphi_o| + |\Delta\lambda_o| \end{cases}$$

Then,

$$\Delta c_{11} d\sin H + \Delta c_{12} d\cos\phi\cos H + \Delta c_{13}(-d\sin\phi\cos H) \le 3d|\Delta\lambda_o| + 2d|\Delta\varphi_o| \tag{18}$$

The following result can be got by using the same method.

$$\begin{cases} \Delta c_{21} d\sin H + \Delta c_{22} d\cos\phi\cos H + \Delta c_{23}(-d\sin\phi\cos H) \le 3d|\Delta\lambda_o| + 2d|\Delta\varphi_o| \\ \Delta c_{31} d\sin H + \Delta c_{32} d\cos\phi\cos H + \Delta c_{33}(-d\sin\phi\cos H) \le 2d|\Delta\varphi_o| \end{cases} \tag{19}$$

For the distance which is less than 1 kilometer, the relationship between *d* and radius of the Earth will satisfy:

$$d << R$$

Where, *R* is the radius of the Earth.

Then,

$$d^2[(\Delta\varphi_o)^2 + (\Delta\lambda_o)^2] << R^2[(\Delta\varphi_o)^2 + (\Delta\lambda_o)^2] \approx [(\Delta x_o^e)^2 + (\Delta y_o^e)^2 + (\Delta z_o^e)^2] \tag{20}$$

That is to say:

$$|\boldsymbol{\Delta}_{12}| << |\boldsymbol{\Delta}_{11}|$$

Thus,

$$\Delta P_1 = |\boldsymbol{\Delta}_1| \approx \sqrt{(\Delta x_o^e)^2 + (\Delta y_o^e)^2 + (\Delta z_o^e)^2} \tag{21}$$

Where, ΔP_1 is the positioning error caused by the positioning error of MEMS/GPS.

(2) Positioning error caused by the error of LDS

Based on the equation (12) and (14), the target positioning error caused by the error of LDS can be calculated with the neglect of other error resources.

$$\boldsymbol{\Delta}_2 = \boldsymbol{C}_e^t \begin{bmatrix} (\tilde{d}-d)\sin H \\ (\tilde{d}-d)\cos\phi\cos H \\ -(\tilde{d}-d)\sin\phi\cos H \end{bmatrix} = \boldsymbol{C}_e^t \begin{bmatrix} \Delta d\sin H \\ \Delta d\cos\phi\cos H \\ -\Delta d\sin\phi\cos H \end{bmatrix}$$

Thus,

$$\Delta P_2 = |\boldsymbol{\Delta}_2| = |\Delta d| \tag{22}$$

(3) Positioning error caused by the attitude error of MEMS/GPS

The target positioning error caused by the attitude error of MEMS/GPS can also be calculated with the neglect of other error resources.

$$\boldsymbol{\Delta}_3 = \boldsymbol{C}_e^t \begin{bmatrix} d(\sin\tilde{H}-\sin H) \\ d(\cos\tilde{\phi}\cos\tilde{H}-\cos\phi\cos H) \\ -d(\sin\tilde{\phi}\cos\tilde{H}-\sin\phi\cos H) \end{bmatrix} = \boldsymbol{C}_e^t \begin{bmatrix} d[\sin(H+\Delta H)-\sin H] \\ d[\cos(\phi+\Delta\phi)\cos(H+\Delta H)-\cos\phi\cos H] \\ -d[\sin(\phi+\Delta\phi)\cos(H+\Delta H)-\sin\phi\cos H] \end{bmatrix} \tag{22}$$

Because both $\Delta\phi$ and ΔH are all small error angles, then

$$\begin{cases} \sin\Delta\phi \approx \Delta\phi & \cos\Delta H \approx 1 & \Delta\phi\cdot\Delta H << \Delta\phi \\ \cos\Delta\phi \approx 1 & \sin\Delta H \approx \Delta H & \Delta\phi\cdot\Delta H << \Delta H \end{cases} \tag{23}$$

Based on the condition of equation (23), the error can be got

$$\boldsymbol{\Delta}_3 = \boldsymbol{C}_e^t \begin{bmatrix} d\Delta H\cos H \\ -d\Delta H\cos\phi\cos H - d\Delta\phi\sin\phi\cos H \\ d\Delta H\sin\phi\sin H - d\Delta\phi\cos\phi\cos H \end{bmatrix} \tag{24}$$

Then,

$$\Delta P_3 = |\boldsymbol{\Delta}_3| = d\sqrt{(\Delta H)^2(\cos^2 H+\cos^2\phi-\cos 2\phi\sin^2 H)+(\Delta\phi)^2\cos^2 H+\Delta H\Delta\phi\sin 2\phi\cos H(\cos H-\sin H)} \tag{25}$$

(4) Simulation for the positioning error of the target

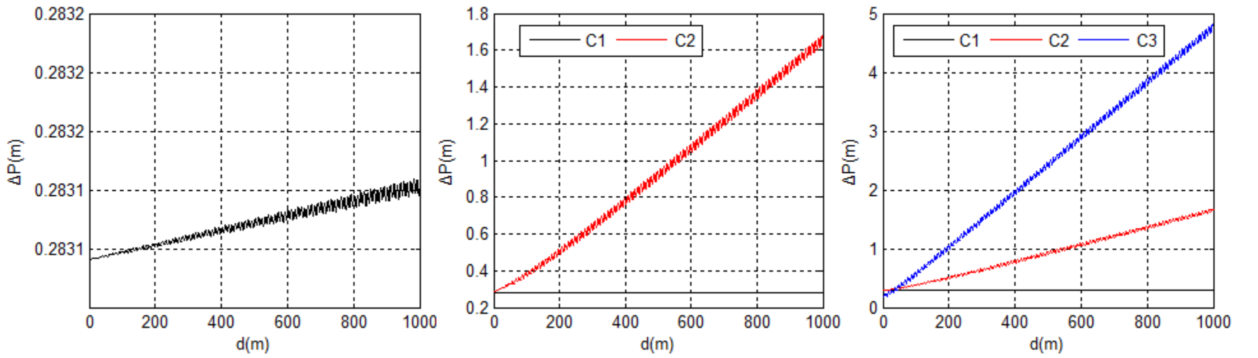
Assume the initial position of the observer is east longitude 126°, north latitude 45°. The longitude and latitude error of MEMS/GPS is 0.2 m, the heading and pitch error of MEMS/GPS are respectively 0.2° and 0.1° [12]. LDS has a linearity error of 0.15% of the range [14]. Corresponding to three conditions, three simulation results are given in Fig.3. By the way, the heading an pitch are in swaying state in order to follow a moving target, $H = 30^\circ + 7^\circ\sin(2\pi * t/7)$, $\phi = 2^\circ + 1^\circ\sin(2\pi * t/7)$.

Condition 1: only the position error of MEMS/GPS is involved;

Condition 2: the position error of MEMS/GPS and the linearity error of LDS are involved;

Condition 3: all errors of MEMS/GPS and LDS are involved.

As shown in Fig. 3(a), positioning error of target *T* caused by the positioning error of MEMS/GPS is mainly concerned with $\boldsymbol{\Delta}_{11}$, because the calculation of $\sqrt{0.2^2+0.2^2}$ is 0.2828, and the error is almost independent of the parameter *d*. The positioning error of the target in Fig.3(b) will reach 1.7m when the parameter *d* is about 1 kilometer. The main reason is the linear error of LDS besides the error of 0.28 m stimulated by condition 1. It also can be seen that the positioning error of the target added quickly after the attitude error is involved in Fig.3(c). In the mean time, the error caused by ΔH and $\Delta\phi$ is linear to the parameter *d*.



(a) Result of condition 1 (b) Results of condition 1, 2 (c) Results of condition 1, 2, 3

FIGURE 3: Simulation of error analysis caused by MEMS/GPS and LDS

4 Calibration for the algorithm

For navigation and mapping with higher accuracy demand, the positioning algorithm probably cannot meet the demands because of the measure error. Former simulation has explained that the reason is because of errors of MEMS/GPS and LDS. Among all errors, attitude error is the most important because it dominates the accuracy of the target position. That is to say, the positioning error of point *o* and the ranging error of *d* can be neglected when the attitude error of MEMS/GPS is bigger than 0.1 deg, especially when the distance *d* is less than 500m.

Besides the attitude error mentioned, the attitude accuracy of MEMS/GPS will decline because the gyro scale factor and the misalignment parameters will change over time [15]. These parameters can only be calibrated in laboratory. Thus, a new calibration method for the system should be designed to improve and maintain the performance of the system.

(1) Description for the known reference point

With the condition of another point (*R*) nearby the observer, its position can be got based on the former algorithm. That is:

$$\begin{bmatrix} \tilde{x}_R^e \\ \tilde{y}_R^e \\ \tilde{y}_R^e \end{bmatrix} = \begin{bmatrix} \tilde{x}_o^e \\ \tilde{y}_o^e \\ \tilde{y}_o^e \end{bmatrix} + \tilde{\boldsymbol{C}}_e^t \begin{bmatrix} \tilde{d}_{oR} \sin \tilde{H} \\ \tilde{d}_{oR} \cos \tilde{\phi} \cos \tilde{H} \\ -\tilde{d}_{oR} \sin \tilde{\phi} \cos \tilde{H} \end{bmatrix} \tag{26}$$

Because error of Δx_o^e, Δy_o^e, Δz_o^e and Δd can be neglected when the parameter *d* is less than 500m. The following equation can be got:

$$\begin{bmatrix} \tilde{x}_R^e \\ \tilde{y}_R^e \\ \tilde{y}_R^e \end{bmatrix} = \begin{bmatrix} x_o^e \\ y_o^e \\ y_o^e \end{bmatrix} + \boldsymbol{C}_e^t \begin{bmatrix} d_{oR} \sin \tilde{H} \\ d_{oR} \cos \tilde{\phi} \cos \tilde{H} \\ -d_{oR} \sin \tilde{\phi} \cos \tilde{H} \end{bmatrix} \tag{27}$$

On the other hand, the ideal position of *R* can be described based on equation (12):

$$\begin{bmatrix} x_R^e \\ y_R^e \\ y_R^e \end{bmatrix} = \begin{bmatrix} x_o^e \\ y_o^e \\ y_o^e \end{bmatrix} + \boldsymbol{C}_e^t \begin{bmatrix} d_{oR} \sin H \\ d_{oR} \cos \phi \cos H \\ -d_{oR} \sin \phi \cos H \end{bmatrix} \tag{28}$$

(2) Attitude calculation based on the known reference point

If the position of the reference can be got in advance by means of MEMS/GPS, the difference of equation (27) and (28) can construct a new measurement variable which can be used to calculate the parameter H and ϕ. That is:

$$\begin{bmatrix}\Delta x_R^e\\ \Delta y_R^e\\ \Delta y_R^e\end{bmatrix}=\begin{bmatrix}\tilde{x}_R^e-x_R^e\\ \tilde{y}_R^e-y_R^e\\ \tilde{y}_R^e-y_R^e\end{bmatrix}=\boldsymbol{C}_e^t\begin{bmatrix}d_{oR}(\sin\tilde{H}-\sin H)\\ d_{oR}(\cos\tilde{\phi}\cos\tilde{H}-\cos\phi\cos H)\\ -d_{oR}(\sin\tilde{\phi}\cos\tilde{H}-\sin\phi\cos H)\end{bmatrix} \tag{29}$$

In equation (29), most variables can be measured by MEMS/GPS and LDS except the ideal attitude angle of ϕ and H. That is to say, angle of $\tilde{\phi}$ and $\tilde{H}$ can be calculated so as to replace the ϕ and H. But, the equation (29) is the type of transcendental equation which is very difficult to solve.In order to get the ideal attitude, equation (29) should be converted into linear equation which can make the solution easy.

The parameter ϕ and H can be substituted as:

$$\begin{cases}\phi=\tilde{\phi}-\Delta\phi\\ H=\tilde{H}-\Delta H\end{cases} \tag{30}$$

Substituting equation (30) into (29), and using the same simplification as equation (23), the transcendental equation can be converted into linear equation.

$$\begin{bmatrix}\Delta x_R^e\\ \Delta y_R^e\\ \Delta y_R^e\end{bmatrix}=\boldsymbol{C}_e^t\begin{bmatrix}d_{oR}\cos\tilde{H}\Delta H\\ -d_{oR}\sin\tilde{\phi}\cos\tilde{H}\Delta\phi-d_{oR}\cos\tilde{\phi}\sin\tilde{H}\Delta H\\ -d_{oR}\cos\tilde{\phi}\cos\tilde{H}\Delta\phi+d_{oR}\sin\tilde{\phi}\sin\tilde{H}\Delta H\end{bmatrix} \tag{31}$$

Equation (31) can also be described as:

$$\begin{bmatrix}\Delta x_R^e\\ \Delta y_R^e\\ \Delta y_R^e\end{bmatrix}=\boldsymbol{C}_e^t\begin{bmatrix}0 & d_{oR}\cos\tilde{H}\\ -d_{oR}\sin\tilde{\phi}\cos\tilde{H} & -d_{oR}\cos\tilde{\phi}\sin\tilde{H}\\ -d_{oR}\cos\tilde{\phi}\cos\tilde{H} & d_{oR}\sin\tilde{\phi}\sin\tilde{H}\end{bmatrix}\begin{bmatrix}\Delta\phi\\ \Delta H\end{bmatrix}=\boldsymbol{C}_e^t\boldsymbol{C}_1\begin{bmatrix}\Delta\phi\\ \Delta H\end{bmatrix} \tag{32}$$

Three equations are redundant for the solution of two unknown variables. Assuming $\boldsymbol{C}=\boldsymbol{C}_e^t\boldsymbol{C}_1$ and involving $\boldsymbol{C}^T$, $\Delta\phi$ and ΔH can be estimated by the least-square method [16].

$$\begin{bmatrix}\Delta\hat{\phi}\\ \Delta\hat{H}\end{bmatrix}=(\boldsymbol{C}^T\cdot\boldsymbol{C})^{-1}\boldsymbol{C}^T\begin{bmatrix}\Delta x_R^e\\ \Delta y_R^e\\ \Delta y_R^e\end{bmatrix} \tag{33}$$

Then, $\hat{\phi}=\tilde{\phi}-\Delta\hat{\phi}$ and $\hat{H}=\tilde{H}-\Delta\hat{H}$ can substitute $\tilde{\phi}$ and $\tilde{H}$. They can improve the attitude of the MEMS/GPS, so as to get the target position with higher accuracy.

5. Simulation for the calibration and the target positioning

(1) Simulation for the calibration

Performance of the calibration is very important for the positioning accuracy of the target. In order to verify the performance of the calibration, simulation results are provided by giving various conditions. During the simulation, all errors of MEMS/GPS and LDS are the same as the former. For the reference point, there is a random error of 0.05m which is influenced by measurement. Six conditions are given during the simulation:

Condition 1: only the attitude error of MEMS/GPS and random error of R are involved, ΔH=0.2°, $\Delta\phi$=0.1°, d_{oR}=200 m;

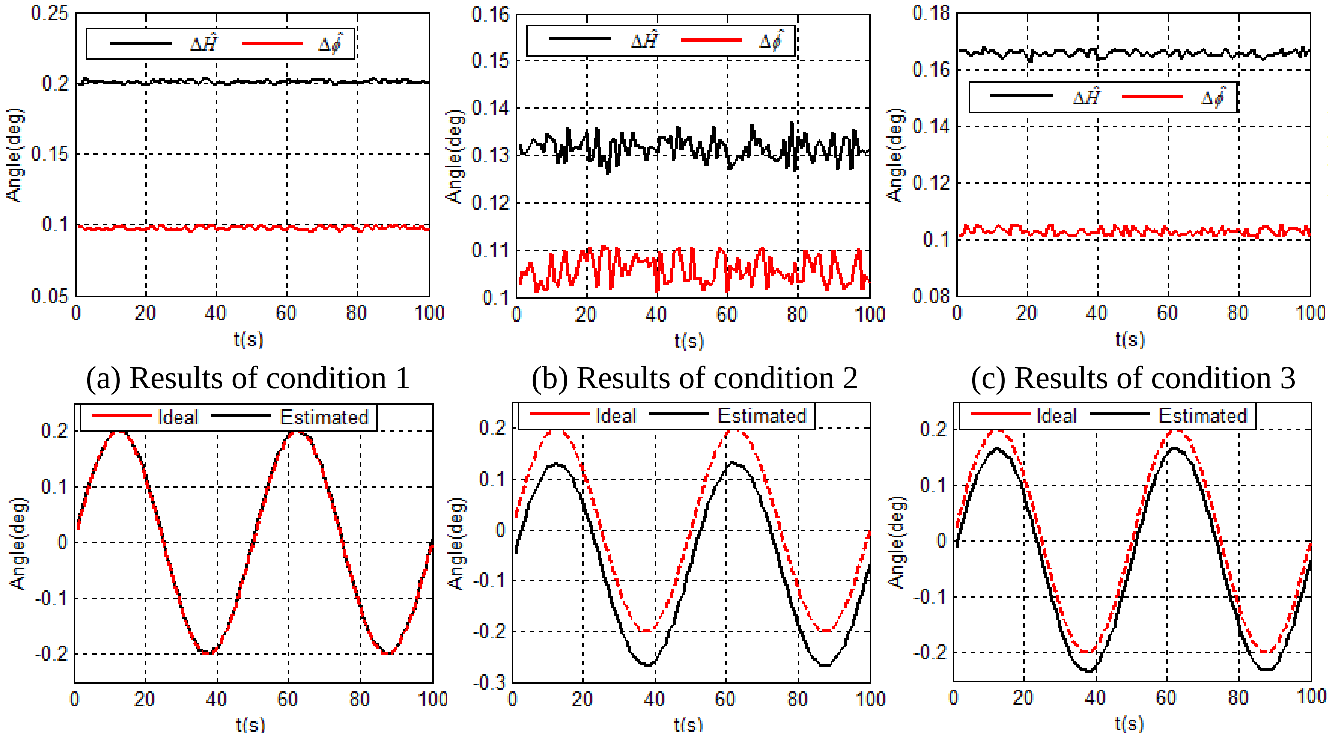
Condition 2: all error of MEMS/GPS, LDS and R are involved, ΔH =0.2°, $\Delta\phi$=0.1°, d_{oR}=200 m;

Condition 3: all error of MEMS/GPS, LDS and R are involved, ΔH =0.2°, $\Delta\phi$=0.1°, d_{oR}=400 m;

Condition 4: only the attitude error of MEMS/GPS and random error of R are involved, $\Delta H = 0.2^{\circ}\sin(2\pi t/50)$, $\Delta\phi = 0.1^{\circ}\sin(2\pi t/50)$, d_{oR}=200 m;

Condition 5: all error of MEMS/GPS, LDS and R are involved, $\Delta H = 0.2^{\circ}\sin(2\pi t/50)$, $\Delta\phi = 0.1^{\circ}\sin(2\pi t/50)$, d_{oR}=200 m;

Condition 6: all error of MEMS/GPS, LDS and R are involved, $\Delta H = 0.2^{\circ}\sin(2\pi t/50)$, $\Delta\phi = 0.1^{\circ}\sin(2\pi t/50)$, d_{oR}=400 m;



(a) Results of condition 1 (b) Results of condition 2 (c) Results of condition 3

(d) Results of condition 4 (e) Results of condition 5 (f) Results of condition 6

FIGURE 4: Performance verification for the calibration algorithm

As shown in Fig. 4(a), the calibration algorithm can estimate the angle of ΔH and $\Delta\phi$, the residual error of ΔH and $\Delta\phi$ are all less than 0.02°, which is a very high accuracy of attitude for the target navigation and mapping. After involving errors of MEMS/GPS and LDS, the accuracy of the calibration decrease because of the error sources. The residual error of ΔH is about 0.07°, and the residual error of $\Delta\phi$ is about 0.01° in Fig. 4(b). But with the increasing of d_{oR}, the residual error of will decrease to 0.04° of ΔH and 0.005° of $\Delta\phi$ (See Fig. 4(c)). That is to say, the performance of the calibration will increase if there is a relatively long distance of d_{oR}. If the type of the angle error is changed, the calibration has the same performance. The comparison of ideal and estimated heading error in Fig. 4(d), (e) and (f) can demonstrate it.

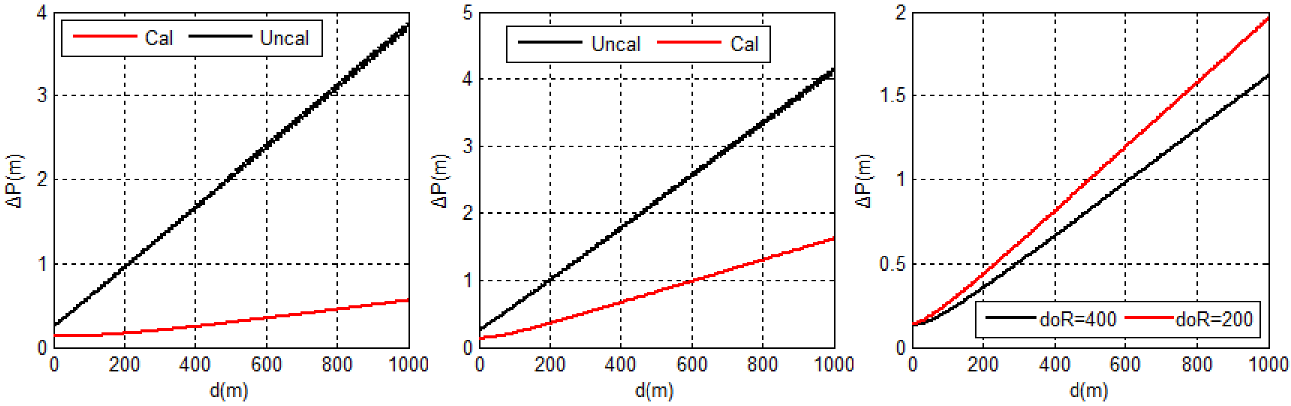
(2) Simulation for the target positioning after calibration

Former simulations demonstrate the calibration algorithm can estimate most attitude errors of MEMS/GPS. This will be very helpful for the accuracy improvement of the target positioning. The following simulations give the final error analysis for the target position after the attitude error of MEMS/GPS is compensated. In order to compare the positioning performance with the un-calibrated algorithm, all errors of MEMS/GPS are the same as the former during the simulation except the error source demanded in the condition.

Condition 1: only the attitude error of MEMS/GPS and random error of R are involved, ΔH =0.2°, $\Delta\phi$=0.1°, d_{oR}=400 m;

Condition 2: all errors of MEMS/GPS, LDS and R are involved, ΔH =0.2°, $\Delta\phi$=0.1°, d_{oR}=400 m;

Condition 3: all errors of MEMS/GPS, LDS and R are involved, ΔH =0.2°, $\Delta\phi$=0.1°, d_{oR}=200 m;



(a) Results of condition 1 (b) Results of condition 2 (c) Results of condition 2 and 3

FIGURE 5: Performance verification for the calibration algorithm

As shown in Fig. 5(a) and (b), the positioning error has been decreased to a large extent after the attitude error of MEMS/GPS is compensated (curve of Cal). For more information in Fig. 5(a) and (b), $\Delta\hat{\phi}$ and $\Delta\hat{H}$ are in fact the equivalent estimation of attitude error which will be influenced by the position error of o and ranging error of d. Because the positioning error of the target is also decreased when the distance is zero, this means that the position error of MEMS/GPS is partly compensated by the calibration algorithm. On the other hand, error of condition 2 is bigger than error of condition 3 because the there are more residual errors when d_{oR}=200m. This perfectly matches with the simulation results of Fig. 4.

6 Conclusions

This study has developed a new method for target navigation and mapping. The results indicate that the proposed method can measure position, height, height difference, slant range and horizontal range for static and moving target. After the calibration of MEMS/GPS, the algorithm can realize precise positioning with the error of less than 2 meters for the static and moving target within 1 kilometer. The stable and superior performances make the new algorithm is very suitable for the target mapping and navigation on land. For application with higher precision demand, linear error LDS should be of calibrated, it is also the further research content of this approach.

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