# Matrix-Based Joint Interference and Channel Order Enumerators for SIMO Systems Suffering From RFI

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***Abstract*—Estimating the number of interferers and their respective channel order is useful for several applications such as the excision of multi-interferer radio frequency interference (RFI) and the mitigation of narrowband interferers prevalent in ultra-wideband communications. For single-input multiple-output systems suffering from multi-interferer RFI, we propose a matrix-based joint enumerator algorithm and its smoothed version to estimate the number of interferers and their respective channel order. Simulations corroborate the joint enumeration capability of the proposed algorithms and an improved performance of the smoothed enumerator which demands higher computation time.**

***Index Terms*—Interference mitigation, source enumeration, channel order estimation, joint estimation.**

## I. Introduction

Source enumeration or model order selection has various applications in wireless communications, biomedical signal processing, geophysical signal processing, array processing, and finance [1]. Hence, it has received considerable attention and several source enumeration algorithms which exploit different criteria have been proposed over the years. To mention some of the main criteria, Rissanen's minimum description length [2], information theoretic criteria [3], and random matrix theory [1], [4], [5]. By employing Rissanen minimum description length, a detection and combined detection-estimation criteria are presented in [2]. Applicable to the enumeration of any kind of sources including fully correlated ones, these criteria are proved to be consistent. In [3], model order selection techniques using Akaike information criterion, generalized information criterion, and Bayesian information criterion are discussed in detail. Apart from these criteria, random matrix theory has recently come into the spotlight for the detection of the number of signals in a white noise. The random matrix theory based approaches [1], [4], [5] are suitable especially for the detection of the number of high dimensional signals contaminated by a white noise in a sample starved setting. It is also reported in [4] that random matrix theory which captures the asymptotic distributions of eigenvalues provides good approximations even for a finite sample size.

On the other hand, channel order estimation or channel identification is a signal processing technique that is critical for several applications. To mention a few applications, multi-user/multi-access communication systems, digital television systems, multi-sensor sonar/radar systems, and speech systems [6]. As a result, several channel order estimation algorithms have been proposed including the recently proposed subspace projection-based algorithm [6]. This algorithm blindly estimates the orders of a finite-duration impulse response (FIR) multiple-input multiple-output (MIMO) and single-input multiple-output (SIMO) systems. Furthermore, a channel order estimation algorithm reported as effective and convenient for SIMO systems exhibiting low or moderate signal-to-noise ratios (SNRs) is proposed in [7].

The multi-linear radio frequency interference (RFI) excision algorithm proposed in [8] requires the estimation of the channel order of the RFI emitted by a single source. The tensor-based multi-interferer RFI (MI-RFI) excision algorithms proposed in [9] and [10] require both the number of interferers and their respective channel order. In this regard, a more accurate joint estimation of the aforementioned parameters renders a much better MI-RFI excision efficiency. Meanwhile, narrowband interference mitigation algorithms [11]–[13] are required for the coexistence of ultra-wideband (UWB) communication systems and nearby devices that render multiple narrowband interferers. Whenever such interferers propagating through multi-path fading channels are received, their efficient mitigation can be executed via the estimation of the number of interferers and their respective channel order. Apart from the aforementioned systems, severe terrestrial MI-RFI propagating through multi-path fading channels can make communication through a mobile satellite system [14], [15] unreliable. The corresponding MI-RFI mitigation, likewise, calls for a joint enumeration of the number of interferers and their respective channel order. Accordingly, it is indispensable to devise a low-complexity joint enumeration algorithm so as to address the aforementioned joint estimation problem encountered in various communication systems. In spite of the fact that a joint enumeration algorithm is required for interference mitigation in various communication systems, such a joint estimation problem is underinvestigated.

In this paper, we propose a **m**atrix-**b**ased **jo**int **n**umber of **i**nterferers and **c**hannel **o**rder **e**numerator (MB-JoNICOE) and a **s**moothed **m**atrix-**b**ased **jo**int **n**umber of **i**nterferers and **c**hannel **o**rder **e**numerator (SMB-JoNICOE). These algorithms employ the eigenvalues of the sample covariance matrix (SCM) and the smoothed SCM (s-SCM), respectively. By computing the singular value decomposition (SVD) of the SCM and s-SCM, these algorithms deploy only one SVD to jointly estimate the number of interferers and their respective

978-15386-3652-7/18/$31.00 ©2018 IEEE

channel order. It is to be noted that the joint estimation is conducted during the first long-term interval while transmitting no signal of interest. To perform joint estimation, the proposed algorithms execute iterative eigenvalue difference and iterative eigenvalue comparison tests which employ adaptive thresholds. Following this introduction, Section II presents the notations and system model. Section III and IV describe the proposed algorithms. Section V reports the simulation results followed by conclusions drawn in Section VI.

## II. Notations and System Model

### A. Notations

Scalars, vectors, and matrices are denoted by italic letters, lower-case boldface letters, and upper-case boldface letters, respectively; $\sim$, $(:,i)$, $||\cdot||_F$, $(\cdot)^T$, and $(\cdot)^H$ implicate distributed as, the $i$th column of a matrix, Frobenius norm, transposition, and Hermitian transposition, respectively; $\text{diag}(\cdot)$, $\min(\cdot)$, $\boldsymbol{I}_{N_R W}$, $\mathbb{E}\{\cdot\}$, $\mathcal{CN}(\cdot,\cdot)$, and $U(\cdot)$ imply diagonal matrix, minimum, $N_R W \times N_R W$ identity matrix, expectation, complex normal distribution, and a unit step function, respectively; $[\boldsymbol{A}, \boldsymbol{B}]$ denotes the horizontal concatenation of $\boldsymbol{A}$ and $\boldsymbol{B}$; $\texttt{diag}(\cdot)$, $\texttt{min}(\cdot)$, $\texttt{length}(\cdot)$, and $\texttt{zeros}(\cdot)$ are the Matlab® functions.

### B. System Model

Consider a SIMO system with $N_R$ receive antennas suffering from MI-RFI emitted by $Q$ independent single-antenna interferers as depicted in Fig. 1. The signal of interest (SOI) channel between the transmitter and each receive antenna pair is modeled as an FIR filter with $L+1$ taps. This channel is assumed to be time-invariant for a long-term interval (LTI). Similarly, the RFI channel between the $i$th RFI transmitter and each receive antenna pair is modeled as an FIR filter with $L_i+1$ taps. For $N_{\text{SOI}}$ being an arbitrary constant, the MI-RFI channel is assumed to have a coherence time of $N_{\text{SOI}}+1$ times the coherence time of the SOI. The received baseband signal at time $n$ then becomes

$$\boldsymbol{y}(n) = \sum_{l=0}^{L} \boldsymbol{h}_l s(n-l) + \sum_{i=1}^{Q} \sum_{l=0}^{L_i} \boldsymbol{g}_i^{(l)} f_i(n-l) + \boldsymbol{z}(n), \quad (1)$$

where $\{\boldsymbol{h}_l, \boldsymbol{g}_i^{(l)}\} \in \mathbb{C}^{N_R}$ are, respectively, the coefficients of the channel impulse responses corresponding to the $l$th SOI and the $i$th RFI's $l$th channel taps, $s(n)$ denotes the symbol emitted by the SOI transmitter at time $n$, $f_i(n)$ is the sampled $i$th broadband RFI which is usually modeled as a zero mean circularly symmetric complex additive white Gaussian noise (AWGN) [16], and $\boldsymbol{z}(n) \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{N_R})$ is a sampled circularly symmetric complex AWGN. Lastly, uncorrelated SOI, $Q$ RFIs, and AWGN are assumed.

## III. MB-JoNICOE Algorithm

As the coherence time of the presumed MI-RFI is greater than the SOI, no SOI is transmitted in the first LTI. During this LTI, the joint enumeration of the number of interferers and their respective channel order is conducted. It is to be

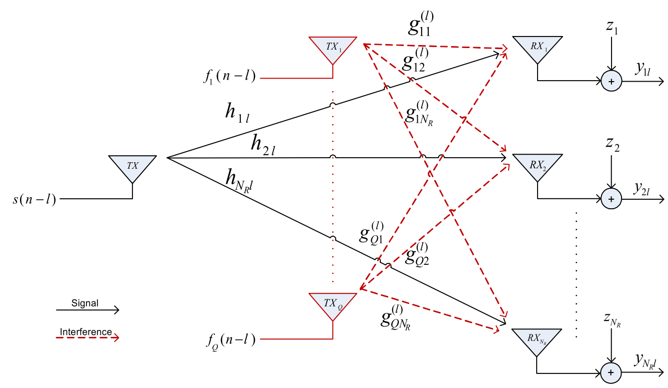


Fig. 1. A baseband schematic depicting the $l$th multi-path component of a SIMO system suffering from $Q$ interferers [10].

noted that an LTI is made of $N$ short-term intervals (STIs). For $T_s$ denoting the symbol duration, a duration equal to $WT_s$ comprises an STI. During each STI, $W$ samples from every $N_R$ antennas are stacked. The horizontal concatenation of $N$ stacked STIs forms a matrix which is exploited by MB-JoNICOE.

The algorithm deploys a single SVD to compute the eigenvalues of the SCM. These eigenvalues are used to jointly enumerate the number of interferers and their respective channel order. In this respect, the algorithm executes iterative eigenvalue difference and iterative eigenvalue comparison tests which employ adaptive thresholds. Iterative eigenvalue difference test with adaptive threshold allows identifying the noise and the MI-RFI eigenvalues. For this test, the initial setting of the adaptive threshold is inspired by the fact that the difference of maximum and minimum noise eigenvalues is zero under infinite samples. Once the noise and MI-RFI eigenvalues are identified, iterative eigenvalue comparison test with adaptive threshold follows. This comparison test is conducted to identify the eigenvalues of each interferer and, in turn, their respective channel order. As a viable channel order is determined for an interferer, the estimated number of interferers increases. Lastly, the MB-JoNICOE algorithm jointly enumerates the number of interferers and their respective channel order.

### A. Problem Setup

With respect to (w.r.t.) the $m$th STI, stacking the observation vectors of the $N_R$ receive antennas and $W$ data windows into one highly structured vector of size $N_R W \times 1$ gives

$$\boldsymbol{y}_m = \boldsymbol{H}\boldsymbol{s}_m + \sum_{i=1}^{Q} \boldsymbol{G}_i \boldsymbol{f}_{im} + \boldsymbol{z}_m \in \mathbb{C}^{N_R W}, \quad (2)$$

where $\boldsymbol{s}_m = \big[s(mW), \ldots, s(mW - W - L + 1)\big]^T \in \mathbb{C}^{(W+L)}$, $\boldsymbol{f}_{im} = \big[f_i(mW), \ldots, f_i(mW - W - L_i + 1)\big]^T \in \mathbb{C}^{(W+L_i)}$, and $\boldsymbol{z}_m$ are the sampled SOI, the $i$th RFI, and a zero mean AWGN, respectively. $\boldsymbol{H} \in \mathbb{C}^{N_R W \times (W+L)}$ is the SOI filtering matrix defined through [17, eqs. (3) and (5)]. $\boldsymbol{G}_i = \big[\boldsymbol{G}_{i1}^T, \ldots, \boldsymbol{G}_{iN_R}^T\big]^T \in \mathbb{C}^{N_R W \times (W+L_i)}$ is the $i$th RFI filtering matrix for $\boldsymbol{G}_{ij} \in \mathbb{C}^{W \times (W+L_i)}$ being a banded

Toeplitz matrix associated with the $i$th RFI and the $j$th receive antenna's impulse response $\boldsymbol{g}_{ij}$. Note that $\boldsymbol{g}_{ij}$ is defined as $\boldsymbol{g}_{ij} \triangleq \left[g_{ij}^0, \ldots, g_{ij}^{L_i}\right]^T = \left[g_{ij}(t_0), \ldots, g_{ij}(t_0 + L_i T_s)\right]^T$, $t_0$ being the time-of-arrival, and $\boldsymbol{G}_{ij}$ is as in [10, eq. (4)].

Expressing the summation in (2) as a matrix product,

$$\boldsymbol{y}_m = \boldsymbol{H}\boldsymbol{s}_m + \boldsymbol{G}\boldsymbol{f}_m + \boldsymbol{z}_m \in \mathbb{C}^{N_R W}, \tag{3}$$

where $\boldsymbol{G} = [\boldsymbol{G}_1, \ldots, \boldsymbol{G}_Q] \in \mathbb{C}^{N_R W \times r}$ is the MI-RFI filtering matrix, for $r = \sum_{i=1}^{Q}(W + L_i)$, and $\boldsymbol{f}_m = [\boldsymbol{f}_{1m}^T, \ldots, \boldsymbol{f}_{Qm}^T]^T \in \mathbb{C}^r$ is the MI-RFI vector. The horizontal concatenation of (3) results in

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{G}\boldsymbol{F} + \boldsymbol{Z} \in \mathbb{C}^{N_R W \times N}, \tag{4}$$

where $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N]$, $\boldsymbol{F} = [\boldsymbol{F}_1^T, \ldots, \boldsymbol{F}_Q^T]^T$, for $\boldsymbol{F}_i = [\boldsymbol{f}_{i1}, \ldots, \boldsymbol{f}_{iN}] \in \mathbb{C}^{(W+L_i)\times N}$, and $\boldsymbol{Z} = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_N]$. In the first LTI, no SOI is transmitted and hence the received signal becomes

$$\boldsymbol{Y}_I = \boldsymbol{G}\boldsymbol{F} + \boldsymbol{Z} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{V}}^H. \tag{5}$$

For the identifiability of the number of interferers and their respective channel order, we assume that $\boldsymbol{F}$ and $\boldsymbol{G}^T$ have full row rank, i.e., $N \geq r$ and $N_R W \geq r$, and $W > \{L_i\}_{i=1}^Q$.

### B. Problem Formulation

The population covariance matrix is obtained by transmitting no SOI in the first LTI as [4, eq. (2)]

$$\boldsymbol{R}_{yy} = \mathbb{E}\{\boldsymbol{y}_m \boldsymbol{y}_m^H\} = \boldsymbol{G}\boldsymbol{R}_{ff}\boldsymbol{G}^H + \sigma^2 \boldsymbol{I}_{N_R W}, \tag{6}$$

where $\boldsymbol{R}_{ff} = \mathbb{E}\{\boldsymbol{f}_m \boldsymbol{f}_m^H\}$ is the MI-RFI covariance matrix. Note that the assumption regarding the uncorrelation of $\boldsymbol{f}_m$ and $\boldsymbol{z}_m$ is exploited. Because each $f_i(n) \sim \mathcal{CN}(0, \sigma_i^2)$, $\boldsymbol{R}_{ff}$ is a diagonal matrix given by

$$\boldsymbol{R}_{ff} = \mathrm{diag}\big(\underbrace{\sigma_1^2, \ldots, \sigma_1^2}_{W+L_1 \text{ terms}}, \underbrace{\sigma_2^2, \ldots, \sigma_2^2}_{W+L_2 \text{ terms}}, \ldots, \underbrace{\sigma_Q^2, \ldots, \sigma_Q^2}_{W+L_Q \text{ terms}}\big), \tag{7}$$

where $\sigma_i^2$ is the power of the $i$th broadband RFI and it is presumed that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_Q$. Using (7) in (6), the first $W + L_1$ eigenvalues might be close to each other, so do the second $W + L_2$ eigenvalues, and so on. From these numbers, we can estimate $\{L_i\}_{i=1}^Q$ and $Q$. However, we can't obtain the population covariance matrix, as infinite samples are required. As a result, we resort to the SCM estimated as [4, eq. (5)]

$$\hat{\boldsymbol{R}}_{yy} = N^{-1}\boldsymbol{Y}_I\boldsymbol{Y}_I^H = N^{-1}\hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{\Sigma}}^H\hat{\boldsymbol{U}}^H = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{U}}^H, \tag{8}$$

where $\hat{\boldsymbol{\Lambda}} = \hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{\Sigma}}^H/N$ comprises the $N_R W$ distinct eigenvalues which can be exploited to identify the noise eigenvalues and the eigenvalues of every interferer. Toward this end, the MB-JoNICOE algorithm is proposed.

### C. The MB-JoNICOE Algorithm

The proposed algorithm is detailed in Algorithm 1. First, this algorithm computes the SVD of $\boldsymbol{Y}_I$ to obtain a vector $\tilde{\boldsymbol{\Lambda}}$ that comprises the eigenvalues of $\boldsymbol{Y}_I$ (lines 1-2). Having employed $\tilde{\boldsymbol{\Lambda}}$, MB-JoNICOE executes iterative eigenvalue difference test which subtracts the minimum eigenvalue $l_{\text{min}}$ from a given eigenvalue so as to determine the noise eigenvalues (lines 5-7). When the aforementioned test generates a value greater than the product of the initialized threshold $\Delta$ and $l_{\text{min}}$ (line 7), the algorithm would preliminarily identify the noise eigenvalues. Hereinafter, it considers the remaining eigenvalues as the MI-RFI eigenvalues and conducts iterative eigenvalue comparison test to render a joint enumeration. In this regard, the algorithm employs the immediate eigenvalue which is greater than the largest estimated noise eigenvalue as a preliminary comparison threshold $l_{\text{th}}$ (line 11 for $c = 0$).

MB-JoNICOE starts an iterative eigenvalue comparison test by comparing the MI-RFI eigenvalues with an adaptive threshold. The adaptive threshold is preliminarily set to $l_{\text{th}}$ plus the value of the $W$th largest eigenvalue w.r.t. the smallest MI-RFI eigenvalue (lines 11 and 16 for $c = 0$). Then, the channel order of the first interferer would be estimated and the eigenvalue comparison test would resume for the remaining interferes (lines 12-20 for $c = 0$) provided that each estimated channel order is less than $W$ (line 18). If not, the loop would break and opt for a smaller comparison threshold by resetting all the estimated channel orders (lines 10-11 for $c > 0$). By the virtue of our assumptions, line 18 ensures that the estimated channel orders are less than $W$. If the loop doesn't break, the number of interferers will be estimated whenever there is a viable channel order estimate for every interferer (lines 17 and 27). Whenever the iterative eigenvalue comparison test resumes by descending $W$ plus the estimated channel order values through $\tilde{\boldsymbol{\Lambda}}$ (line 19), MB-JoNICOE will make sure that the last largest eigenvalue under test is the closest to the maximum eigenvalue (lines 21-24).

When iterative eigenvalue difference test and iterative eigenvalue comparison test satisfy all the loop controls, the algorithm returns the number of interferers and their respective channel order (line 27).

## IV. Smoothed MB-JoNICOE: SMB-JoNICOE

Defined as the number of new samples in the next observed data window, smoothing the received signal with a smoothing factor $\eta$, $1 \leq \eta < W$, provides more observations. These additional observations are provided through overlapping observation windows at the expense of computation time. Such a smoothing operation has improved the performance of the tensor-based channel estimation algorithm proposed in [17]. Similarly, we exploit per-antenna overlapping windows to propose SMB-JoNICOE which enhances MB-JoNICOE.

### A. Problem Setup

If $\eta$ new samples are included in the subsequent STIs, the observation windows will overlap for $1 \leq \eta < W$. Smoothing (4) with such $N^s$ overlapping windows gives

$$\boldsymbol{Y}^s = \boldsymbol{H}\boldsymbol{S}^s + \boldsymbol{G}\boldsymbol{F}^s + \boldsymbol{Z}^s \in \mathbb{C}^{N_R W \times N^s}, \tag{9}$$

where $\boldsymbol{S}^s = \left[\boldsymbol{s}_1^s, \ldots, \boldsymbol{s}_{N^s}^s\right]$, $\boldsymbol{s}_m^s = \left[s\big(W + (m-1)\eta\big), \ldots, s\big(W + (m-1)\eta - W - L + 1\big)\right]^T \in \mathbb{C}^{(W+L)}$, $\boldsymbol{F}^s = [\boldsymbol{F}_{1s}^T, \ldots, \boldsymbol{F}_{Qs}^T]^T \in \mathbb{C}^{r \times N^s}$ for $\boldsymbol{F}_{is} = [\boldsymbol{f}_{i1}^s, \ldots, \boldsymbol{f}_{iN^s}^s]$ and $\boldsymbol{f}_{im}^s = \big[f_i\big(W + (m-1)\eta\big), \ldots, f_i\big(W + (m-1)\eta -$

**Algorithm 1:** MB-JoNICOE Algorithm

**Input**: $\boldsymbol{Y}_I$, $N_R$, $W$, $N$
**Output**: $\{\hat{L}(\hat{Q}-i)\}_{i=0}^{\hat{Q}-1}$, $\hat{Q}$
1 Set values for $\Delta$, $\xi$; decomposition of $\boldsymbol{Y}_I$ as in (5)
2 $\tilde{\boldsymbol{\Lambda}} = \mathtt{diag}\big(\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{\Sigma}}^H\big)/N$; $l_{\min} = \mathtt{min}\big(\tilde{\boldsymbol{\Lambda}}\big)$
3 **repeat**
4 $\quad$ $\Delta \leftarrow \xi\Delta$; $k = \mathtt{length}(\tilde{\boldsymbol{\Lambda}})$
5 $\quad$ **repeat**
6 $\quad\quad$ $\hat{r} \leftarrow k$; $k \leftarrow k-1$
7 $\quad$ **until** $\tilde{\boldsymbol{\Lambda}}(k) - l_{\min} \geq l_{\min}\Delta$;
8 $\quad$ $m \leftarrow k - W$; $c \leftarrow 0$
9 $\quad$ **repeat**
10 $\quad\quad$ **if** $\hat{r} + c > \mathtt{length}(\tilde{\boldsymbol{\Lambda}})$, *then* **break**
11 $\quad\quad$ $l_{\text{th}} \leftarrow \tilde{\boldsymbol{\Lambda}}(\hat{r}+c)$; $\hat{Q} \leftarrow 0$; $\hat{L} \leftarrow \mathtt{zeros}(1, 100)$; $k \leftarrow \hat{r} - 1$; $m \leftarrow k - W$
12 $\quad\quad$ **repeat**
13 $\quad\quad\quad$ $\hat{l} = 0$
14 $\quad\quad\quad$ **repeat**
15 $\quad\quad\quad\quad$ $\hat{l} \leftarrow \hat{l} + 1$; $k \leftarrow k - 1$
16 $\quad\quad\quad$ **until** $\tilde{\boldsymbol{\Lambda}}(k) \geq \tilde{\boldsymbol{\Lambda}}(m) + l_{\text{th}}$ & $k \geq 1$;
17 $\quad\quad\quad$ $\hat{Q} \leftarrow \hat{Q} + 1$
18 $\quad\quad\quad$ **if** $\hat{l} - W \geq W$, *then* **break**
19 $\quad\quad\quad$ $\hat{L}(\hat{Q}) = \hat{l} - W$; $m \leftarrow m - W - \hat{L}(\hat{Q})$
20 $\quad\quad$ **until** $m < 1$;
21 $\quad\quad$ **if** $m < 0$, *then* **break**
22 $\quad\quad$ $c \leftarrow c + 1$
23 $\quad$ **until** $m < 0$;
24 $\quad$ **if** $m < 0$, *then* **break**
25 $\quad$ $\xi \leftarrow \xi + 1$
26 **until** $\xi\Delta \geq 2$;
27 **return** $\hat{L}(\hat{Q})$, $\hat{L}(\hat{Q}-1)$, …, $\hat{L}(2)$, $\hat{L}(1)$, $\hat{Q}$

$W - L_f^i + 1)\big]^T \in \mathbb{C}^{(W+L_f^i)}$, and $\boldsymbol{Z}^s$ is the smoothed AWGN matrix. Transmitting no SOI in the first LTI,

$$\boldsymbol{Y}_I^s = \boldsymbol{G}\boldsymbol{F}^s + \boldsymbol{Z}^s = \hat{\boldsymbol{U}}^s\hat{\boldsymbol{\Sigma}}^s\hat{\boldsymbol{V}}^{s^H} \in \mathbb{C}^{N_R W \times N^s}, \tag{10}$$

For the identifiability of the parameters to be estimated, we adopt the assumptions stated in Section III-A.

### B. Problem Formulation

Similarly, the s-SCM is computed using (10) as

$$\hat{\boldsymbol{R}}_{y^s y^s} = (N^s)^{-1}\big[\boldsymbol{Y}_I^s\boldsymbol{Y}_I^{s^H} = \hat{\boldsymbol{U}}^s\hat{\boldsymbol{\Sigma}}^s\hat{\boldsymbol{\Sigma}}^{s^H}\hat{\boldsymbol{U}}^{s^H}\big] = \hat{\boldsymbol{U}}^s\hat{\boldsymbol{\Lambda}}^s\hat{\boldsymbol{U}}^{s^H}, \tag{11}$$

where $\hat{\boldsymbol{\Lambda}}^s = \hat{\boldsymbol{\Sigma}}^s\hat{\boldsymbol{\Sigma}}^{s^H}/N^s$ consists of the distinct eigenvalues which can be employed to identify the eigenvalues of the noise and every RFI. Using these eigenvalues, the number of interferers and their respective channel order are estimated.

### C. The SMB-JoNICOE Algorithm

The SMB-JoNICOE algorithm, which exploits the eigenvalues of the s-SCM, is detailed in Algorithm 2. As stated, this algorithm also relies on iterative eigenvalue difference and iterative eigenvalue comparison tests. Whenever there is a viable channel order estimate, the estimated number of interferers increases. Eventually, the number of interferers and their respective channel order would be jointly enumerated.

**Algorithm 2:** SMB-JoNICOE Algorithm

**Input**: $\boldsymbol{Y}_I^s$, $N_R$, $W$, $N^s$
**Output**: $\{\hat{L}(\hat{Q}-i)\}_{i=0}^{\hat{Q}-1}$, $\hat{Q}$
Set values for $\Delta$, $\xi$; decomposition of $\boldsymbol{Y}_I^s$ as in (10)
$\tilde{\boldsymbol{\Lambda}} = \mathtt{diag}\big(\hat{\boldsymbol{\Sigma}}^s\hat{\boldsymbol{\Sigma}}^{s^H}\big)/N^s$; $l_{\min} = \mathtt{min}\big(\tilde{\boldsymbol{\Lambda}}\big)$
**repeat**
$\quad$ Execute lines 4-25 of Algorithm 1
**until** $\xi\Delta \geq 2$;
**return** $\hat{L}(\hat{Q})$, $\hat{L}(\hat{Q}-1)$, …, $\hat{L}(2)$, $\hat{L}(1)$, $\hat{Q}$

| Simulation parameters | Assigned value |
|---|---|
| $[L_1, L_2, L_3]$ | $[1, 1, 1]$ |
| $[\sigma_1, \sigma_2, \sigma_3]$ | $[1, 1, 1]$ W |
| $[t_0, \eta]$ | $[0.1T_s, 1]$ |
| $[\Delta, \beta, \xi]$ | $[0.05, 0.5, 1]$ |
| No. of channel realizations | 1000 |

TABLE I
SIMULATION PARAMETERS UNLESS OTHERWISE MENTIONED.

## V. SIMULATION RESULTS

Using the parameters of Table I, unless otherwise mentioned, the performance of MB-JoNICOE and SMB-JoNICOE are simulated as per Algorithms 1 and 2, respectively. During the first LTI, $Q$ zero mean circularly symmetric complex white Gaussian signals, as a broadband MI-RFI, are transmitted over multi-path fading channels. To simulate the $i$th RFI's multi-path fading channels, $(L_i + 1)$-ray multi-path continuous-time channels are constructed synchronously using the raised cosine pulse shaping filter $p_{rc}(t, \beta)$ exhibiting a roll-off factor $\beta$ as $g_{ij}(t) = \sum_{l=0}^{L_i} g_{ij}^l p_{rc}(t - lT_s, \beta)$, for $g_{ij}^l \sim \mathcal{CN}(0, 1)$ [17], [18]. For $\boldsymbol{G}_i$ normalized to a Frobenius norm of $\sqrt{W}$, the interference-to-noise ratio (INR), in dB, is defined as $\gamma_{inr} = 10\log_{10}\mathbb{E}\{\|\boldsymbol{G}\boldsymbol{F}\|_F^2\}/\mathbb{E}\{\|\boldsymbol{Z}\|_F^2\}$. The performance of the proposed enumerators is assessed via a joint root mean square error (J-RMSE) defined as J-RMSE $= U(-\Delta Q)\sqrt{\mathbb{E}\{\sum_{i=1}^{\hat{Q}}\Delta(L_i, Q)\}} + U(\Delta Q)\sqrt{\mathbb{E}\{\sum_{i=1}^{Q}\Delta(L_i, Q)\}}$, for $\Delta(L_i, Q) = (\Delta L_i)^2 + (\Delta Q)^2$, $\Delta Q = \hat{Q} - Q$, and $\Delta L_i = \hat{L}(i) - L_i$.

Fig. 2 depicts the J-RMSE performance of MB-JoNICOE and SMB-JoNICOE for $Q = 2$, different INR, and the same $N_{\text{tot}}$ denoting the number of observed symbols per LTI. As observed in Fig. 2, the proposed algorithms result in a lower J-RMSE w.r.t. the increment of INR. This improvement happens for the fact that high INR enables a better identification of the eigenvalues of the noise and the MI-RFI than a small or moderate INR. Moreover, SMB-JoNICOE outperforms MB-JoNICOE as the INR increases, since smoothing results in a more overlapping observation windows and hence a better parameter estimation.

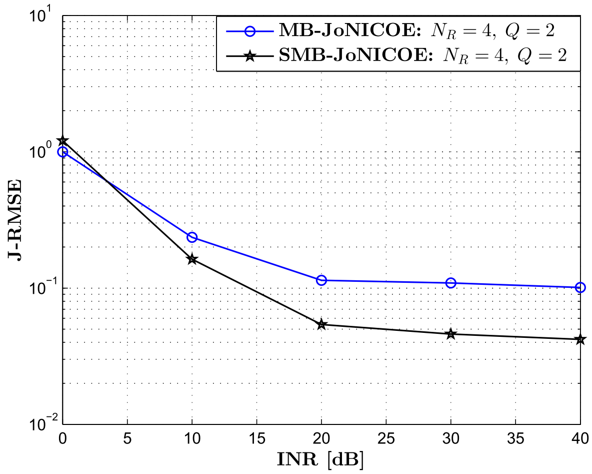


Fig. 2. J-RMSE performance of MB-JoNICOE and SMB-JoNICOE: $N_{\text{tot}} = 1080$ and $W = 3$.

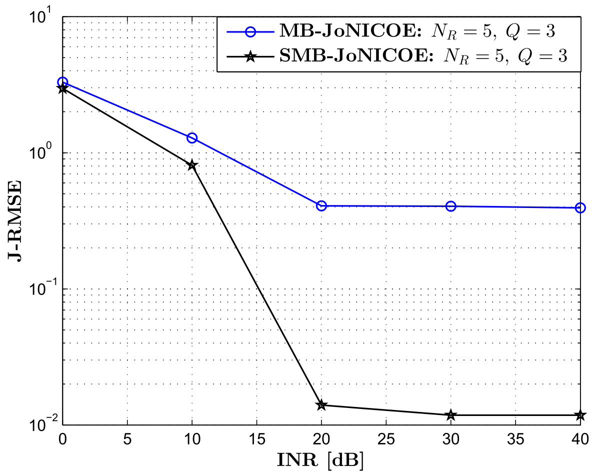


Fig. 3. J-RMSE performance of MB-JoNICOE and SMB-JoNICOE: $N_{\text{tot}} = 8000$ and $W = 5$.

Fig. 3 corroborates the J-RMSE performance of MB-JoNICOE and SMB-JoNICOE for $Q = 3$, the same $N_{\text{tot}}$, and different INR. It is evident that SMB-JoNICOE outperforms MB-JoNICOE as the INR gets larger. This significant performance gain is attributed to smoothing which offers more overlapping observation windows. Moreover, it is demonstrated in the same figure that a strong MI-RFI has a better quality joint estimates than the weak one, as the eigenvalues of the former are easier to identify than the latter. Comparing Figs. 2 and 3, increasing $Q$ requires more samples for a proper joint enumeration.

## VI. Conclusions

We propose two matrix-based enumerators named MB-JoNICOE and SMB-JoNICOE that exploit the eigenvalues of the SCM and s-SCM, respectively. Using these eigenvalues, the proposed algorithms perform iterative eigenvalue difference and iterative eigenvalue comparison tests which deploy adaptive thresholds. The iterative eigenvalue difference test enables the identification of the eigenvalues of the noise and of the interferers, whereas the iterative eigenvalue comparison test possibly identifies the eigenvalues of each interferer and, in turn, their respective channel order. Whenever there is a feasible channel order estimate, the estimated number of interferers increases. Monte-Carlo simulations corroborate the joint enumeration capability of the proposed algorithms. Although at the cost of computation time, simulations also demonstrate that SMB-JoNICOE improves MB-JoNICOE.

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