Matrix-Based Joint Interference and Channel Order Enumerators for SIMO Systems Suffering From RFI

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***Abstract*—Estimating the number of interferers and their respective channel order is important for several applications such as the excision of multi-interferer radio frequency interference (RFI) and the mitigation of narrowband interferers prevalent in ultra-wideband communications. In this work, we propose a matrix-based joint enumerator algorithm and its smoothed version in order to estimate the number of interferers and their respective channel order for single-input multiple-output systems suffering from multi-interferer RFI. The proposed algorithms rely on iterative eigenvalue difference and iterative eigenvalue comparison tests that deploy adaptive threshold. Monte-Carlo simulations corroborate the joint enumeration capability of the proposed algorithms and an improved performance of the smoothed enumerator which demands higher computation time.**

***Index Terms*—Interference mitigation, source enumeration, channel order estimation, joint estimation.**

I. Introduction

Source enumeration or model order selection has various applications in wireless communications, biomedical signal processing, geophysical signal processing, array processing, and finance [1]. Hence, it has received considerable attention and several source enumeration algorithms which exploit different criteria have been proposed over the years. To mention some of the main criteria, Rissanen's minimum description length [2], information theoretic criteria [3], and random matrix theory [1] [4] [5]. By employing Rissanen minimum description length, a detection and combined detection-estimation criteria are presented in [2]. Applicable to the enumeration of any kind of sources including fully correlated ones, these criteria are proved to be consistent. In [3], model order selection techniques using Akaike information criterion, generalized information criterion, and Bayesian information criterion are discussed in detail. Apart from these criteria, random matrix theory has recently come into the spotlight for the detection of the number of signals in a white noise. The random matrix theory based approaches [1] [4] [5] are suitable especially for the detection of the number of high dimensional signals contaminated by a white noise in a sample starved setting. It is also reported in [4] that random matrix theory which captures the asymptotic distributions of the eigenvalues provides good approximations even for a finite sample size.

On the other hand, channel order estimation or channel identification is a signal processing problem which exists in several applications. To mention a few, multi-user/multi-access communication systems, digital television systems, multi-sensor sonar/radar systems, and speech systems [6]. As a result, it has received considerable attention and several channel order estimation algorithms have been proposed including the recently proposed subspace projection-based algorithm [6]. This algorithm blindly estimates the orders of a finite-duration impulse response (FIR) multiple-input multiple-output (MIMO) and single-input multiple-output (SIMO) systems. Furthermore, a channel order estimation algorithm reported as effective and convenient for SIMO systems exhibiting low or moderate signal-to-noise ratios (SNRs) is proposed in [7]. The work in [7] is based on combining a monotonically decreasing blind channel identification cost function and a monotonically increasing blind channel equalization cost function.

The multi-linear radio frequency interference (RFI) excision algorithm proposed in [8] demands the estimation of the channel order of the RFI emitted by a single source. The tensor-based multi-interferer RFI (MI-RFI) excision algorithm which is proposed in [9] requires both the number of interferers and their respective channel order. In this regard, a more accurate joint estimation of the aforementioned parameters render a much better MI-RFI excision efficiency. Meanwhile, narrowband mitigation algorithms [10] [11] [12] are required for the coexistence of ultra-wideband (UWB) communication systems and nearby devices that render multiple narrowband interferers. Whenever such interferers propagating through multi-path fading channels are received, their efficient mitigation might be executed via the estimation of the number of interferers and their respective channel order. Apart from the aforementioned systems, severe terrestrial MI-RFI propagating through multi-path fading channels can make communication through a mobile satellite system [13] [14] unreliable. The corresponding MI-RFI mitigation, likewise, calls for a joint enumeration of the number of interferers and their respective channel order. Accordingly, it is indispensable to devise a low-complexity joint enumeration algorithm so as to address the aforementioned joint estimation problem encountered in various communication systems.

In spite of the fact that a joint enumeration algorithm is required for interference mitigation in various communication systems, such a joint estimation has not been addressed to date to the best of our knowledge. Meanwhile, the algorithm proposed in [6] estimates the number of sources of a MIMO system via the number of subsystems that attain each channel order. However, it can't identify the respective channel order of each source and is very complex in terms of the number of singular value decompositions (SVDs) that should be computed.

In this contribution, we propose a **m**atrix-**b**ased **jo**int **n**umber of **i**nterferers and **c**hannel **o**rder **e**numerator (MB-JoNICOE) and a **s**moothed **m**atrix-**b**ased **jo**int **n**umber of **i**nterferers and **c**hannel **o**rder **e**numerator (SMB-JoNICOE). These algorithms employ the eigenvalues of the sample covariance matrix (SCM) and the smoothed SCM (s-SCM), respectively. By computing the SVD of the SCM and s-SCM, these algorithms deploy only one SVD to jointly estimate the number of interferers and their respective channel order. It is to be noted that the joint estimation is conducted during the first long-term interval while transmitting no signal of interest. To perform joint estimation, the proposed algorithms execute iterative eigenvalue difference and iterative eigenvalue comparison tests which employ adaptive thresholds. Following this introduction, Section II presents the notations and system model. Section III and IV detail the proposed algorithms. Section V reports the simulation results followed by conclusions drawn in Section VI.

II. Notations and System Model

A. Notations

Throughout the paper, scalars, vectors, and matrices are denoted as italic letters, lowercase boldface letters, and uppercase boldface letters, respectively. The notations $\sim$, $(:,i)$, $||\cdot||_F$, $(\cdot)^T$, $(\cdot)^H$, diag$(\cdot)$, min$(\cdot)$, $\boldsymbol{I}_{N_RW}$, $\mathbb{E}\{\cdot\}$, $\mathcal{CN}(\cdot,\cdot)$, and $U(\cdot)$ imply distributed as, the ith column of a matrix, Frobenius norm, transposition, Hermitian transposition, diagonal matrix, minimum, $N_RW \times N_RW$ identity matrix, expectation, complex normal distribution, and unit step function, respectively. Meanwhile, the horizontal concatenation of $\boldsymbol{A}$ and $\boldsymbol{B}$ is denoted as $[\boldsymbol{A}, \boldsymbol{B}]$. Furthermore, `diag`$(\cdot)$, `min`$(\cdot)$, `length`$(\cdot)$, and `zeros`$(\cdot)$ are the Matlab® functions.

B. System Model

We consider a SIMO system with N_R receive antennas suffering from severe MI-RFI emitted by Q independent single-antenna interferers as shown in Fig. 1. The signal of interest (SOI) channel between the transmitter and each receive antenna pair is modeled as an FIR filter with $L+1$ taps. This channel is assumed to be time-invariant for a long-term interval (LTI). Similarly, the RFI channel between the ith RFI transmitter and each receive antenna pair is modeled as an FIR filter with $L_i + 1$ taps. For N_{SOI} being an arbitrary constant, the MI-RFI channel is assumed to have a coherence time of $N_{\text{SOI}} + 1$ times the coherence time of the SOI. The received baseband signal at time n then becomes

$$\boldsymbol{y}(n) = \sum_{l=0}^{L} \boldsymbol{h}_l s(n-l) + \sum_{i=1}^{Q}\sum_{l=0}^{L_i} \boldsymbol{g}_i^{(l)} f_i(n-l) + \boldsymbol{z}(n), \quad (1)$$

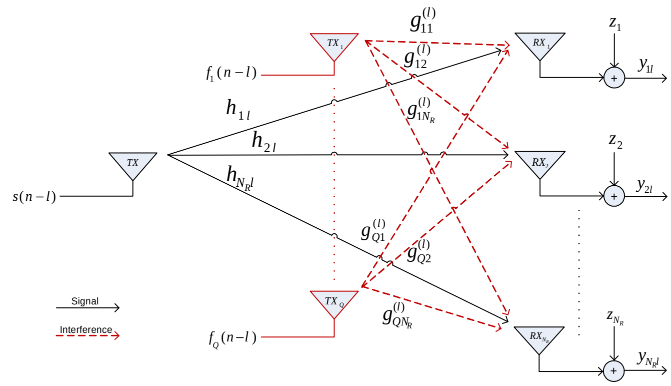

Fig. 1. A baseband schematic depicting the lth multi-path component of a SIMO system suffering from Q interferers.

where $\{\boldsymbol{h}_l, \boldsymbol{g}_i^{(l)}\} \in \mathbb{C}^{N_R}$ are, respectively, the coefficients of the channel impulse responses corresponding to the lth SOI and the ith RFI's lth channel taps, $s(n)$ denotes the symbol emitted by the SOI transmitter at time n, $f_i(n)$ is the sampled ith broadband RFI which is usually modeled as a zero mean circularly symmetric complex additive white Gaussian noise (AWGN) [15], and $\boldsymbol{z}(n) \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2\boldsymbol{I}_{N_R})$ is a sampled circularly symmetric complex AWGN. Furthermore, we assume that the SOI, the Q RFIs, and the AWGN are uncorrelated.

III. MB-JoNICOE Algorithm

As the coherence time of the presumed MI-RFI is greater than the SOI, no SOI is transmitted in the first LTI. During this LTI, the joint enumeration of the number of interferers and their respective channel order is executed. It is to be noted that an LTI is made of N short-term intervals (STIs). For T_s being the symbol duration, the duration of WT_s comprises an STI. During each STI, W samples from every N_R antennas are stacked. The horizontal concatenation of N stacked STIs forms a matrix which is exploited by MB-JoNICOE.

The algorithm deploys a single SVD to compute the eigenvalues of the SCM. These eigenvalues are used to jointly enumerate the number of interferers and their respective channel order. In this respect, the algorithm executes iterative eigenvalue difference and iterative eigenvalue comparison tests which employ adaptive thresholds. Iterative eigenvalue difference test with adaptive threshold allows identifying the noise and the MI-RFI eigenvalues. For this test, the initial setting of the adaptive threshold is inspired by the fact that the difference of maximum and minimum noise eigenvalues is zero under infinite samples. Once the noise and MI-RFI eigenvalues are identified, iterative eigenvalue comparison test with adaptive threshold follows. This comparison test is conducted to identify the eigenvalues of each interferer and in turn their respective channel order. As a viable channel order is determined for an interferer, the estimated number of interferers would increase. Lastly, the MB-JoNICOE algorithm jointly enumerates the number of interferers and their respective channel order. Hereinafter, the algorithm preceded by its problem setup and formulation is presented.

A. Problem Setup

With respect to (w.r.t) the mth STI, stacking the observation vectors of the N_R receive antennas and W data windows into one highly structured vector of size $N_R W \times 1$ gives

$$\boldsymbol{y}_m = \boldsymbol{H}\boldsymbol{s}_m + \sum_{i=1}^{Q} \boldsymbol{G}_i \boldsymbol{f}_{im} + \boldsymbol{z}_m \in \mathbb{C}^{N_R W}, \quad (2)$$

where $\boldsymbol{s}_m = \big[s\big(mW\big), \ldots, s\big(mW - W - L + 1\big)\big]^T \in \mathbb{C}^{(W+L)}$, $\boldsymbol{f}_{im} = \big[f_i\big(mW\big), \ldots, f_i\big(mW - W - L_i + 1\big)\big]^T \in \mathbb{C}^{(W+L_i)}$, and $\boldsymbol{z}_m$ are the sampled SOI, ith RFI, and a zero mean AWGN, respectively. $\boldsymbol{H} \in \mathbb{C}^{N_R W \times (W+L)}$ is the SOI filtering matrix defined through [16, eq. (3) & (5)]. $\boldsymbol{G}_i = \big[\boldsymbol{G}_{i1}^T, \ldots, \boldsymbol{G}_{iN_R}^T\big]^T \in \mathbb{C}^{N_R W \times (W+L_i)}$ is the ith RFI filtering matrix for $\boldsymbol{G}_{ij} \in \mathbb{C}^{W \times (W+L_i)}$ being a banded Toeplitz matrix associated with the ith RFI and the jth receive antenna's impulse response $\boldsymbol{g}_{ij}$. $\boldsymbol{g}_{ij}$ is defined as $\boldsymbol{g}_{ij} \triangleq \big[g_{ij}^0, \ldots, g_{ij}^{L_i}\big]^T = \big[g_{ij}(t_0), \ldots, g_{ij}(t_0 + L_i T_s)\big]^T$, where t_0 is the time-of-arrival, and

$$\boldsymbol{G}_{ij} = \begin{bmatrix} g_{ij}^0 & \ldots & g_{ij}^{L_i} & 0 & \ldots & \ldots & 0 \\ 0 & g_{ij}^0 & \ldots & g_{ij}^{L_i} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & \ldots & 0 & g_{ij}^0 & \ldots & g_{ij}^{L_i} \end{bmatrix}. \quad (3)$$

Expressing the summation in (2) as a matrix product renders

$$\boldsymbol{y}_m = \boldsymbol{H}\boldsymbol{s}_m + \boldsymbol{G}\boldsymbol{f}_m + \boldsymbol{z}_m \in \mathbb{C}^{N_R W}, \quad (4)$$

where $\boldsymbol{G} = [\boldsymbol{G}_1, \ldots, \boldsymbol{G}_Q] \in \mathbb{C}^{N_R W \times r}$ is the MI-RFI filtering matrix, for $r = \sum_{i=1}^{Q}(W + L_i)$, and $\boldsymbol{f}_m = [\boldsymbol{f}_{1m}^T, \cdots, \boldsymbol{f}_{Qm}^T]^T \in \mathbb{C}^r$ is the MI-RFI vector. The horizontal concatenation of (4) then results in

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{G}\boldsymbol{F} + \boldsymbol{Z} \in \mathbb{C}^{N_R W \times N}, \quad (5)$$

where $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N]$, $\boldsymbol{F} = [\boldsymbol{F}_1^T, \ldots, \boldsymbol{F}_Q^T]^T$, for $\boldsymbol{F}_i = [\boldsymbol{f}_{i1}, \ldots, \boldsymbol{f}_{iN}] \in \mathbb{C}^{(W+L_i) \times N}$, and $\boldsymbol{Z} = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_N]$. In the first LTI, no SOI is transmitted and hence the received signal becomes

$$\boldsymbol{Y}_I = \boldsymbol{G}\boldsymbol{F} + \boldsymbol{Z} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{V}}^H, \quad (6)$$

where $\hat{\boldsymbol{U}} \in \mathbb{C}^{N_R W \times N_R W}$, $\hat{\boldsymbol{\Sigma}} \in \mathbb{C}^{N_R W \times N}$, and $\hat{\boldsymbol{V}} \in \mathbb{C}^{N \times N}$. For the identifiability of the number of interferers and their respective channel order, we assume that $\boldsymbol{F}$ and $\boldsymbol{G}^T$ have full row rank, i.e., $N \geq r$ and $N_R W \geq r$. Besides, we assume that $W > \{L_i\}_{i=1}^{Q}$.

B. Problem Formulation

The space-time autocorrelation matrix or the population covariance matrix is obtained by transmitting no SOI in the first LTI as [4, eq. (2)]

$$\boldsymbol{R}_{yy} = \mathbb{E}\{\boldsymbol{y}_m \boldsymbol{y}_m^H\} = \boldsymbol{G}\boldsymbol{R}_{ff}\boldsymbol{G}^H + \sigma^2 \boldsymbol{I}_{N_R W}, \quad (7)$$

where $\mathbb{E}\{\cdot\}$ implies expectation and $\boldsymbol{R}_{ff} = \mathbb{E}\{\boldsymbol{f}_m \boldsymbol{f}_m^H\}$ is the MI-RFI covariance matrix. Note that the assumption regarding the uncorrelation of $\boldsymbol{f}_m$ and $\boldsymbol{z}_m$ is exploited. Because each $f_i(n) \sim \mathcal{CN}(0, \sigma_i^2)$, $\boldsymbol{R}_{ff}$ is a diagonal matrix given by

$$\boldsymbol{R}_{ff} = \text{diag}\Big(\underbrace{\sigma_1^2, \ldots, \sigma_1^2}_{W+L_1 \text{ terms}}, \underbrace{\sigma_2^2, \ldots, \sigma_2^2}_{W+L_2 \text{ terms}}, \ldots, \underbrace{\sigma_Q^2, \ldots, \sigma_Q^2}_{W+L_Q \text{ terms}}\Big), \quad (8)$$

where σ_i^2 is the power of the ith broadband RFI and it is presumed that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_Q$. Employing (8) into (7), the first $W + L_1$ eigenvalues are identical, so do the second $W + L_2$ eigenvalues and so on. From theses numbers, we can obtain $\{L_i\}_{i=1}^{Q}$ and Q. However, we can't obtain the population covariance matrix—as infinite samples are required—and hence we resort to the estimation of the SCM obtained as [4, eq. (5)]

$$\hat{\boldsymbol{R}}_{yy} = \frac{1}{N}\boldsymbol{Y}_I\boldsymbol{Y}_I^H = \frac{1}{N}\hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{\Sigma}}^H\hat{\boldsymbol{U}}^H = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{U}}^H, \quad (9)$$

where $\hat{\boldsymbol{\Lambda}} = \hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{\Sigma}}^H/N$ and $l_1 > l_2 > \ldots > l_r > \ldots > l_{N_R W}$ are the distinct eigenvalues of the SCM and $N_R W - r$ of them are contributed by the AWGN. Using these eigenvalues, the noise eigenvalues and the eigenvalues of each interferer can be identified for the joint enumeration. Toward this end, the MB-JoNICOE algorithm is devised.

C. The MB-JoNICOE Algorithm

The proposed algorithm is detailed in Algorithm 1. First, this algorithm computes the SVD of $\boldsymbol{Y}_I$ to obtain a vector $\tilde{\boldsymbol{\Lambda}}$ that comprises the eigenvalues of $\boldsymbol{Y}_I$ (lines 1-2). Having employed $\tilde{\boldsymbol{\Lambda}}$, MB-JoNICOE executes iterative eigenvalue difference test which subtracts the minimum eigenvalue l_{min} from a given eigenvalue so as to determine the noise eigenvalues (lines 5-7). When the aforementioned test generates a value greater than the product of the initialized threshold Δ and l_{min} (line 7), the algorithm would preliminarily identify the noise eigenvalues. Hereinafter, it considers the remaining eigenvalues as the MI-RFI eigenvalues and conducts iterative eigenvalue comparison test to render a joint enumeration. In this regard, the algorithm employs the immediate eigenvalue that is greater than the largest estimated noise eigenvalue as a preliminary comparison threshold l_{th} (line 11 for $c = 0$).

MB-JoNICOE starts an iterative eigenvalue comparison test by comparing the MI-RFI eigenvalues with an adaptive threshold. The adaptive threshold is preliminarily set to l_{th} plus the value of the Wth largest eigenvalue w.r.t the smallest MI-RFI eigenvalue (lines 11 and 16 for $c = 0$). Then, the channel order of the first interferer would be estimated and the eigenvalue comparison test would resume for the remaining interferes (lines 12-20 for $c = 0$) provided that each estimated channel order is less than W (line 18). If not, the loop would break and opt for a smaller comparison threshold by resetting all the estimated channel orders (lines 10-11 for $c > 0$). By the virtue of our assumptions, line 18 ensures that the estimated channel orders are less than W. If the loop doesn't break, the number of interferers will be estimated whenever there is a viable channel order estimate for every interferer (lines 17 and 27). Whenever the iterative eigenvalue comparison test resumes by descending W plus the estimated channel order values through $\tilde{\mathbf{\Lambda}}$ (line 19), MB-JoNICOE will make sure that the last largest eigenvalue under test is the closest to the maximum eigenvalue (lines 21-24).

When iterative eigenvalue difference test and iterative eigenvalue comparison test satisfy all the loop controls, the algorithm returns the number of interferers and their respective channel order (line 27).

Algorithm 1: MB-JoNICOE Algorithm

Input: $\boldsymbol{Y}_I$, N_R, W, N
Output: $\{\hat{L}(\hat{Q}-i)\}_{i=0}^{\hat{Q}-1}$, $\hat{Q}$
1 Set values for Δ, ξ; decomposition of $\boldsymbol{Y}_I$ as in (6)
2 $\tilde{\mathbf{\Lambda}} = \mathtt{diag}\big(\hat{\mathbf{\Sigma}}\hat{\mathbf{\Sigma}}^H\big)/N$; $l_{\min} = \mathtt{min}\big(\tilde{\mathbf{\Lambda}}\big)$
3 **repeat**
4 $\quad$ $\Delta \leftarrow \xi\Delta$; $k = \mathtt{length}(\tilde{\mathbf{\Lambda}})$
5 $\quad$ **repeat**
6 $\quad\quad$ $\hat{r} \leftarrow k$; $k \leftarrow k-1$
7 $\quad$ **until** $\tilde{\mathbf{\Lambda}}(k) - l_{\min} \geq l_{\min}\Delta$;
8 $\quad$ $m \leftarrow k - W$; $c \leftarrow 0$
9 $\quad$ **repeat**
10 $\quad\quad$ **if** $\hat{r} + c > \mathtt{length}(\tilde{\mathbf{\Lambda}})$, *then* **break**
11 $\quad\quad$ $l_{\text{th}} \leftarrow \tilde{\mathbf{\Lambda}}(\hat{r}+c)$; $\hat{Q} \leftarrow 0$; $\hat{L} \leftarrow \mathtt{zeros}(1, 100)$; $k \leftarrow \hat{r} - 1$; $m \leftarrow k - W$
12 $\quad\quad$ **repeat**
13 $\quad\quad\quad$ $\hat{l} = 0$
14 $\quad\quad\quad$ **repeat**
15 $\quad\quad\quad\quad$ $\hat{l} \leftarrow \hat{l} + 1$; $k \leftarrow k - 1$
16 $\quad\quad\quad$ **until** $\tilde{\mathbf{\Lambda}}(k) \geq \tilde{\mathbf{\Lambda}}(m) + l_{\text{th}}$ & $k \geq 1$;
17 $\quad\quad\quad$ $\hat{Q} \leftarrow \hat{Q} + 1$
18 $\quad\quad\quad$ **if** $\hat{l} - W \geq W$, *then* **break**
19 $\quad\quad\quad$ $\hat{L}(\hat{Q}) = \hat{l} - W$; $m \leftarrow m - W - \hat{L}(\hat{Q})$
20 $\quad\quad$ **until** $m < 1$;
21 $\quad\quad$ **if** $m < 0$, *then* **break**
22 $\quad\quad$ $c \leftarrow c + 1$
23 $\quad$ **until** $m < 0$;
24 $\quad$ **if** $m < 0$, *then* **break**
25 $\quad$ $\xi \leftarrow \xi + 1$
26 **until** $\xi\Delta \geq 2$;
27 **return** $\hat{L}(\hat{Q})$, $\hat{L}(\hat{Q}-1)$, $\ldots$, $\hat{L}(2)$, $\hat{L}(1)$, $\hat{Q}$

IV. Smoothed MB-JoNICOE: SMB-JoNICOE

Smoothing the received signal with a smoothing factor η, $1 \leq \eta < W$, defined as the number of new samples in the next observed data window provides more observations. These additional observations are provided through overlapping observation windows at the expense of computation time. Such a smoothing operation has improved the performance of the tensor-based channel estimation algorithm proposed in [16]. Likewise, we exploit per-antenna overlapping windows to propose SMB-JoNICOE which enhances MB-JoNICOE.

A. Problem Setup

If η new samples are included in the subsequent STIs, the observation windows will overlap for $1 \leq \eta < W$. Smoothing (5) with such N^s overlapping windows gives

$$\boldsymbol{Y}^s = \boldsymbol{H}\boldsymbol{S}^s + \boldsymbol{G}\boldsymbol{F}^s + \boldsymbol{Z}^s \in \mathbb{C}^{N_R W \times N^s}, \tag{10}$$

where $\boldsymbol{S}^s = \big[\boldsymbol{s}_1^s, \ldots, \boldsymbol{s}_{N^s}^s\big]$, $\boldsymbol{s}_m^s = \big[s\big(W + (m-1)\eta\big), \ldots, s\big(W + (m-1)\eta - W - L + 1\big)\big]^T \in \mathbb{C}^{(W+L)}$, $\boldsymbol{F}^s = [\boldsymbol{F}_{1s}^T, \ldots, \boldsymbol{F}_{Qs}^T]^T \in \mathbb{C}^{r \times N^s}$ for $\boldsymbol{F}_{is} = [\boldsymbol{f}_{i1}^s, \ldots, \boldsymbol{f}_{iN^s}^s]$ and $\boldsymbol{f}_{im}^s = \big[f_i\big(W + (m-1)\eta\big), \ldots, f_i\big(W + (m-1)\eta - W - L_f^i + 1\big)\big]^T \in \mathbb{C}^{(W+L_f^i)}$, and $\boldsymbol{Z}^s$ is the smoothed AWGN matrix.

Like MB-JoNICOE, SMB-JoNICOE transmits no SOI in the first LTI. Accordingly, the smoothed received signal becomes

$$\boldsymbol{Y}_I^s = \boldsymbol{G}\boldsymbol{F}^s + \boldsymbol{Z}^s = \hat{\boldsymbol{U}}^s\hat{\mathbf{\Sigma}}^s\hat{\boldsymbol{V}}^{s^H} \in \mathbb{C}^{N_R W \times N^s}, \tag{11}$$

where $\hat{\boldsymbol{U}}^s \in \mathbb{C}^{N_R W \times N_R W}$, $\hat{\mathbf{\Sigma}}^s \in \mathbb{C}^{N_R W \times N^s}$, and $\hat{\boldsymbol{V}}^s \in \mathbb{C}^{N^s \times N^s}$. Meanwhile, we adopt the assumptions of Section III-A for the sake of the identifiability of the number of interferers and their respective channel order.

B. Problem Formulation

Similarly, the s-SCM is computed using (11) as

$$\hat{\boldsymbol{R}}_{y^s y^s} = \frac{1}{N^s}\boldsymbol{Y}_I^s\boldsymbol{Y}_I^{s^H} = \frac{1}{N^s}\hat{\boldsymbol{U}}^s\hat{\mathbf{\Sigma}}^s\hat{\mathbf{\Sigma}}^{s^H}\hat{\boldsymbol{U}}^{s^H} = \hat{\boldsymbol{U}}^s\hat{\mathbf{\Lambda}}^s\hat{\boldsymbol{U}}^{s^H}, \tag{12}$$

where $\hat{\mathbf{\Lambda}}^s = \hat{\mathbf{\Sigma}}^s\hat{\mathbf{\Sigma}}^{s^H}/N^s$ and $l_1^s > \ldots > l_r^s > \ldots > l_{N_R W}^s$ denote the distinct eigenvalues of the s-SCM. By employing these eigenvalues, the eigenvalues of the noise and every RFI would be identified. Through the identified eigenvalues, the number of interferers and their respective channel order can be determined as elucidated by the next subsection.

C. The SMB-JoNICOE Algorithm

The SMB-JoNICOE algorithm which exploits the eigenvalues of the s-SCM is detailed in Algorithm 2. Likewise, the algorithm follows identical routines as Algorithm 1 and relies on iterative eigenvalue difference and iterative eigenvalue comparison tests. Whenever there is a viable channel order estimate, the estimated number of interferers would increase—like MB-JoNICOE. Eventually, the number of interferers and their respective channel order would be jointly enumerated.

Algorithm 2: SMB-JoNICOE Algorithm

Input: $\boldsymbol{Y}_I^s$, N_R, W, N^s
Output: $\{\hat{L}(\hat{Q}-i)\}_{i=0}^{\hat{Q}-1}$, $\hat{Q}$
Set values for Δ, ξ; decomposition of $\boldsymbol{Y}_I^s$ as in (11)
$\tilde{\mathbf{\Lambda}} = \mathtt{diag}\big(\hat{\mathbf{\Sigma}}^s\hat{\mathbf{\Sigma}}^{s^H}\big)/N^s$; $l_{\min} = \mathtt{min}\big(\tilde{\mathbf{\Lambda}}\big)$
repeat
$\quad$ Execute lines 4-25 of Algorithm 1
until $\xi\Delta \geq 2$;
return $\hat{L}(\hat{Q})$, $\hat{L}(\hat{Q}-1)$, $\ldots$, $\hat{L}(2)$, $\hat{L}(1)$, $\hat{Q}$

Simulation parameters	Assigned value
$[L_1, L_2, L_3]$	$[1, 1, 1]$
$[\sigma_1, \sigma_2, \sigma_3]$	$[1, 1, 1]$ W
$[t_0, \eta]$	$[0.1T_s, 1]$
$[\Delta, \beta, \xi]$	$[0.05, 0.5, 1]$
No. of channel realizations	1000

TABLE I
SIMULATION PARAMETERS UNLESS OTHERWISE MENTIONED.

V. SIMULATION RESULTS

This section presents the performance of MB-JoNICOE and SMB-JoNICOE which are simulated using the parameters of Table I unless otherwise mentioned. The conducted simulations deploy the succeeding simulation setup.

During the first LTI, Q zero mean circularly symmetric complex white Gaussian signals—as a broadband MI-RFI—are transmitted over multi-path fading channels. To simulate the ith RFI multi-path fading channels, $(L_i + 1)$-ray multi-path continuous-time channels are constructed synchronously using the raised cosine pulse shaping filter $p_{rc}(t, \beta)$ exhibiting a roll-off factor β as $g_{ij}(t) = \sum_{l=0}^{L_i} g_{ij}^l p_{rc}(t - lT_s, \beta)$, for $g_{ij}^l \sim \mathcal{N}(0, 1)$ [16] [17]. For $\boldsymbol{G}_i$ normalized to a Frobenius norm of $\sqrt{W}$, the interference-to-noise ratio (INR) in dB, denoted as γ_{inr}, is defined as

$$\gamma_{inr} = 10 \log_{10} \frac{\mathbb{E}\Big\{ \|\boldsymbol{GF}\|_F^2 \Big\}}{\mathbb{E}\Big\{ \|\boldsymbol{Z}\|_F^2 \Big\}}. \tag{13}$$

The performance of the proposed enumerators is assessed via joint root mean square error (J-RMSE) defined as

$$\text{J-RMSE} = U(-\Delta Q) \sqrt{\mathbb{E}\Big\{ \sum_{i=1}^{\hat{Q}} (\Delta L_i)^2 + (\Delta Q)^2 \Big\}} + U(\Delta Q) \sqrt{\mathbb{E}\Big\{ \sum_{i=1}^{Q} (\Delta L_i)^2 + (\Delta Q)^2 \Big\}}, \tag{14}$$

where $\Delta Q = \hat{Q} - Q$, $\Delta L_i = \hat{L}(i) - L_i$, and $U(\cdot)$ implies a unit step function. The Monte-Carlo simulations of MB-JoNICOE and SMB-JoNICOE are conducted as per Algorithm 1 and Algorithm 2, respectively. These simulations result in the plots depicted subsequently.

Fig. 2 depicts the J-RMSE performance of MB-JoNICOE and SMB-JoNICOE for $Q = 2$, different INR, and the same N_{tot}—being the number of observed symbols per LTI. As observed in Fig. 2, the proposed algorithms result in a J-RMSE performance improvement w.r.t the increment of INR. This improvement happens for the fact that high INR enables a better identification of the eigenvalues of the noise and the MI-RFI than a small or moderate INR. Moreover, SMB-JoNICOE outperforms MB-JoNICOE as the INR increases, since smoothing results in more overlapping observation windows and hence a better parameter estimation.

Fig. 3 corroborates the J-RMSE performance of MB-JoNICOE and SMB-JoNICOE for $Q = 2$, the same N_{tot}, and different INR. It is evident that SMB-JoNICOE outperforms MB-JoNICOE as the INR gets larger. This significant performance gain is attributed to the smoothing which offers more overlapping observation windows. Moreover, it is demonstrated in the same figure that a strong MI-RFI would have better joint estimates than the weak one, as the eigenvalues of the former are easier to identify than the latter. Comparing Figs. 2 and 3, increasing Q requires an increment of N_{tot} for a proper joint enumeration.

Fig. 4 corroborates the J-RMSE performance of MB-JoNICOE and SMB-JoNICOE for $Q = 3$, different N_{tot}, and fixed INR. As depicted, the performance of MB-JoNICOE and SMB-JoNICOE improves as the number of observed symbols increases. The improvement is because of the fact that the eigenvalues of the noise and every RFI are better identified whenever the number of observed symbols increases. Similarly, SMB-JoNICOE with $\eta = 1$ outperforms MB-

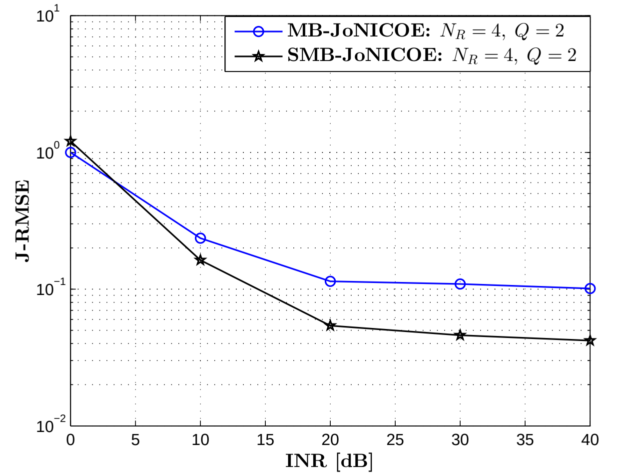

Fig. 2. J-RMSE performance of MB-JoNICOE and SMB-JoNICOE: $N_{\text{tot}} = 1080$ and $W = 3$.

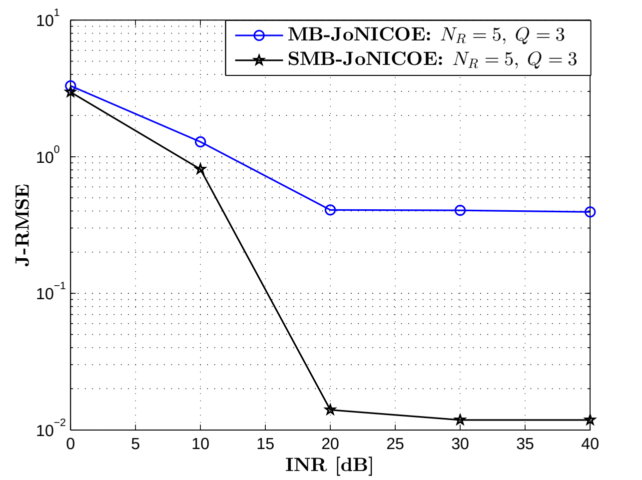

Fig. 3. J-RMSE performance of MB-JoNICOE and SMB-JoNICOE: $N_{\text{tot}} = 8000$ and $W = 5$.

JoNICOE especially for high N_{tot}, since smoothing, through more observations, results in an improved identification of the eigenvalues of the noise and every RFI.

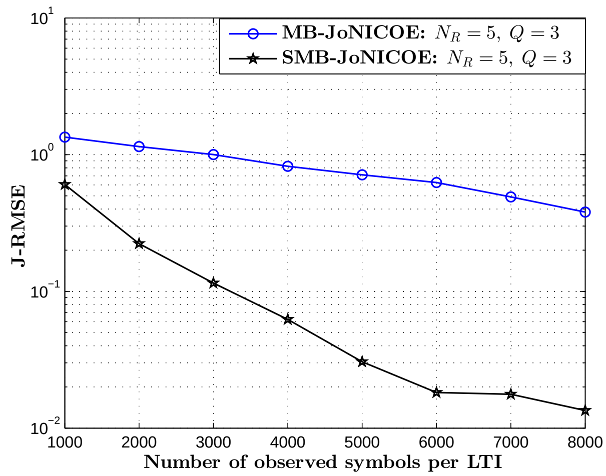

Fig. 4. J-RMSE performance of MB-JoNICOE and SMB-JoNICOE: $\gamma_{inr} =$ 30 dB and $W = 5$.

VI. Conclusions

The joint enumeration of the number of interferers and their respective channel order finds applications in several communication systems. In this work, we propose two matrix-based enumerators named MB-JoNICOE and SMB-JoNICOE. These algorithms, respectively, exploit the eigenvalues of the SCM and s-SCM. To obtain the required eigenvalues, the proposed algorithms transmit no signal of interest at the beginning and perform one singular value decomposition on the vertically stacked and concatenated matrix made by the samples of the N_R antennas. Next, the proposed algorithms conduct iterative eigenvalue difference and iterative eigenvalue comparison tests which deploy adaptive threshold. The iterative eigenvalue difference test enables the identification of the eigenvalues of the noise and of the interferences, whereas the iterative eigenvalue comparison test identifies the eigenvalues of each interferer and in turn their respective channel order. Whenever there is a viable channel order estimate, the estimated number of interferers would increase.

Monte-Carlo simulations corroborate the joint enumeration capability of the proposed algorithms. Simulations also demonstrate that SMB-JoNICOE significantly improves the performance of MB-JoNICOE in terms of J-RMSE through more number of observations. Nevertheless, the performance improvement comes at the cost of computation time. Having witnessed the eigenvalues of the sample covariance matrix and its smoothed version performing the required joint enumeration, we envision that this would inspire further research on the eigenvalue spread of a given space-time autocorrelation matrix.

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