Fuzzy Enhancement of GPS – INS Synergy

Adrian Hiliuta, René Jr. Landry, François Gagnon,
Ecole de technologie superieure, *Montreal, Canada*

BIOGRAPHY

Adrian Hiliuta received the B.Eng. (1990) and Ph.D. (2001) degrees in aerospace engineering from Military Technical Academy, Bucharest, Romania.
René Jr. Landry received a Bachelor Engineering Degree at the Ecole Polytechnique of Montreal, Canada in 1992, a Master of Science in Satellite Communication Engineering at the University of Surrey, Guildford, England in 1993, a Master in Space Electronics and a DEA in Microwaves at the ENSAE/ SupAero, Toulouse, France in 1994.
François Gagnon (S'87-M'87-SM'99) received the B.Eng. and Ph.D. degrees in electrical engineering from École Polytechnique de Montréal, Montréal, P.Q., Canada.

ABSTRACT

The paper presents a new approach of enhancement of GPS – INS synergy by the use of a fuzzy system. A new membership function, which allows a better management of its shape, is used in Mamdani-type fuzzy inference systems. There are two main parameters in fuzzy control systems that influence the performances of a system. The first parameter is the overlapping of membership functions and the second is the output value after defuzzification. Both of them are related to the shape of membership functions. This is modified either by the change of classical membership functions or by the use of concentrator and dilator operators. We introduce a novel membership function which has an elliptical shape. This elliptical membership function provides for the same operation effects, but with much more accuracy. Essentially, the new membership function uses one parameter, the curvature, to obtain a flexible and useful set of functions. Furthermore, the curvature parameter is intuitively appealing. The theoretical fundamentals of the function are presented and the practical accuracy aspects are evaluated. A feed forward system using two Mamdani-type fuzzy inference subsystems is built, and then trained by the modification of the shapes of membership functions.

INTRODUCTION

During the past few years there are approaches focusing on using artificial intelligence to cope with problems arising from incertitude in hybrid navigation systems. The necessity of greater integrity of the navigation solution leads to approaches that do not put all eggs in the same basket. Although the GNSS is the most popular positioning system, when it comes to integrity most of the approaches use also the Inertial Navigation System (INS) as a backup. The synergy of GNSS and INS is the basis of the hybrid navigation systems. Loosely or tightly coupled, the two systems offer a solution for almost every customer. There is a huge literature on Kalman filtering of GPS and INS separately, when they are loosely coupled, or on GPS an INS together when they are tightly coupled.

David McNeil Mayhew uses fuzzy techniques to overcome some problems in a loosely coupled hybrid system [1]. His approach is to weigh the GPS and INS outputs in function of the dynamic scenario. The weights are computed using fuzzy logic in different scenarios. For instance, if the vehicle speed is low the heading computed based on GPS fixes is not accurate and the weight is transferred to INS.

Two years later, Escamilla-Ambrosio and Mort used a Fuzzy-adaptive Kalman Filter to adjust the measurement noise covariance matrix R employing a fuzzy inference system (FIS) [2].

In 2002, Rahbari, Leach, Dillon and de Silva approached the same problem of noise covariance matrix and tune this matrix, using fuzzy techniques, as function of aircraft maneuvers [3].

In [4] Loebis, Sutton and Chudley presents a very good review of multisensor data fusion and the papers published to date on artificial intelligence fusion using neural networks, fuzzy sets and genetic algorithms. The same authors will explore, one year later [5], a hybrid navigation system, which cope with the Kalman filter divergence problem caused by the insufficiently known filter statistics. The filter is adapted by use of fuzzy-rule-based scheme. Genetic algorithm techniques are then used to optimize the parameters of membership functions.

A team of researchers from Department of Geomatics Engineering, University of Calgary, designed an adaptive fuzzy network to modify the Kalman filter that tightly coupled the GPS and INS [6].

The authors of the present paper explored in [7] the possibility of use of predetermined scenarios to train an adaptive network fuzzy inference system. The system would eventually learn about the INS errors while GPS is available and try to apply the structured knowledge about these errors to correct them while GPS is unavailable.

A similar approach using pure neural network was proposed by Wang and Gao in [8].

As we can see there are two main approaches in using artificial intelligence for INS and GPS integration. Similar to use of Kalman filter, we can say that there is a tightly coupled integration, when the artificial intelligence is used to adapt a Kalman filter that computes navigation solution based on both INS and GPS data, and a loosely coupled integration when separate solutions from GPS and INS are used into a system which compensates their errors.

The paper presents an approach that loosely integrates GPS with INS by use of human reasoning, implemented in two Mamdani-type fuzzy inference systems. The first one acts as a filter for GPS navigation solution and, basically, implements the following reasoning: "If there is a big change in GPS solution that is not found in INS solution, then there is a big possibility that the change is not real". This reasoning exploits the short time stability of INS and is called Short-time Estimator. The second fuzzy system is used to predict the long term INS error. The error is computed as the difference between the GPS position and INS position averaged over 30 seconds. In this manner, the long term GPS accuracy is enhanced. The estimate of INS error for next 30 seconds is computed using the current computed error, for the last 30 seconds, and the previous variation of this error. The predicted error is used to correct the Short-time Estimator output in a feed forward correction system.

To achieve these goals we needed a new membership function to be used in designing the fuzzy inference systems. The paragraph II presents a recall of Mamdani-type fuzzy inference systems and introduces the possibility to change the shape of membership functions, as well as the overlapping of these functions. In the paragraph III, a new membership function is developed. The paragraph IV presents details of the hybrid navigation system. The last paragraph presents the conclusions.

MODIFICATION OF THE MEMBERSHIP DEGREE OF THE EXISTING MEMBERSHIP FUNCTION

Instead of complicated differential equations, fuzzy systems work with linguistic logic and structured knowledge [9]. Therefore, the fuzzy models of real systems are closer to human reasoning, being useful where uncertainty appears, and this happens almost everywhere in the real world [10].

Fuzzy systems essentially differ from stochastic or deterministic ones. They use the degree of uncertainty rather than probability or differential equations. The fuzzy logic derives from multivalent logic. The main concept of fuzzy logic is that the membership of an element in a set is a matter of degree. When we say an affirmation as "the speed is fast", that is a matter of degree. If we think that "fast" means 30 m/s, then what is a speed of 29 m/s? What about 28 m/s? We can say that the values 28 m/s and 29 m/s are close enough to 30 m/s and they are "fast" too. However, as value, 30 is larger than 28, so 30 m/s is a "fast" speed in a larger degree than 28. If we define also, a "very fast" speed, then 32 m/s is a "very fast" speed in a larger degree than 30 m/s, but is a "fast" speed in a smaller degree than 30 m/s. Representing the variation of degree of membership, using a linear dependency, leads to a shape as in Fig. 1. This is a classical membership function (mf), named triangular mf. There are, also, trapezoidal, parabolic, Gaussian or singleton mfs. Each of them makes a link between a value of a deterministic variable and the degree of membership in a fuzzy set, as is "fast" for the variable "speed". We say, in this case, that "speed" is a linguistic variable, and the value "fast" is its linguistic degree. All the linguistic degrees (i.e. "fast", "slow", "very fast" etc.) for a linguistic variable represent the universe of discourse of that variable.

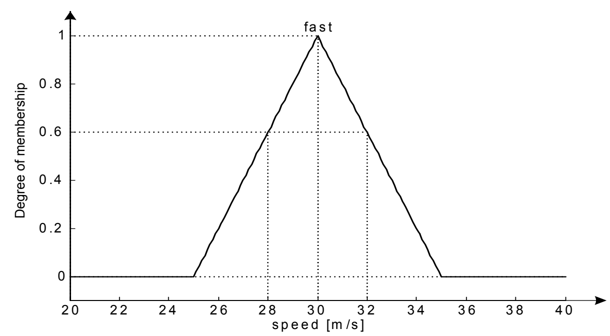

Fig. 1. The degree of membership for the membership function "fast" of the linguistic variable "speed".

The Fig. 1 shows that the degree of membership of the value *speed* = 28 m/s, in the fuzzy set "fast", is 0.6. However, for specific applications the variation of degree of membership can be non-linear. In this case, other mfs, more complicated, are used. If, in an application, a specific MF is used and one has to change its shape, in order to modify the degree of membership of a linguistic variable, this can be done using the concentrator and dilator operators.

The concentrator operator, applied to a membership function m, is defined by [11]

$$\text{conc}(m) = m^2 . \quad (1)$$

This operator can be applied twice and we get

$$\text{conc}(\text{conc}(m)) = m^4 . \quad (2)$$

The modification of membership degree is depicted in Fig. 2.

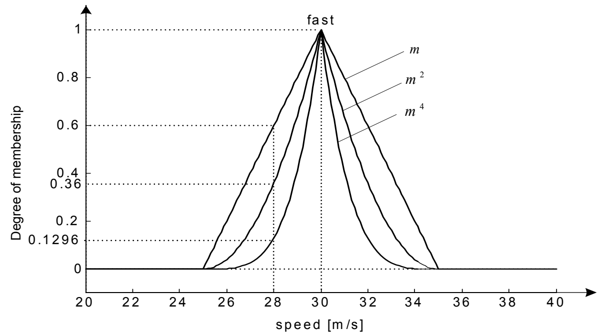

Fig. 2. The modification of the membership degree through concentrator operator.

In the same way, the dilator operator is defined by

$$\mathrm{dil}(m)=\sqrt{m}\,, \tag{3}$$

and its application, once and twice, is depicted in Fig. 3.

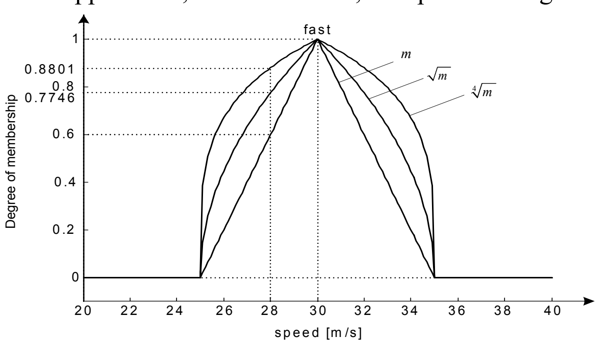

Fig. 3. The modification of the membership degree through dilator operator.

The modification of the shape can be done also using real values for the exponent, with for example 1.623. This approach leads to complicated expressions and to a longer time for evaluation. In this manner, the advantage of using fuzzy systems (due to simple mathematical operations) can be lost.

The approach in this paper presents a new mf, named elliptical membership function, which permits the modification of the membership degree and of the overlapping of MFs in continuous way, by changing only one parameter.

This membership function came from the use of the fuzzy logic in the fuzzy integration between two navigation systems: Global Positioning System and Inertial Navigation System. Such approaches are made, without the use of elliptical mf, by NASA for fuzzy navigational state estimator [12] and for the autonomous navigation of planetary rovers [13], as well as at Flight Research Laboratory, in Canada, for fuzzy logic adaptive tuning in an integrated navigator [14].

ANALYTICAL EXPRESSION OF THE ELLIPTICAL MEMBERSHIP FUNCTION

The Geometrical Transformation of the Space of Membership Function

The Fig. 2 and 3 show the transformation of the linear function in a non-linear one. The same effect can be obtained using circular arcs or, more general, elliptical arcs. The basic idea is to start with the linear mf, as the simplest one, and to curve it, increasing or decreasing the degree of membership. The modification of the curvature from 0 to 1 will modify the shape from straight line to maximum bending possible. The use of the elliptical arcs will allow us to replace the effects of both concentrator and dilator operators, by defining positive and negative curvature.

Considering the definition interval of the MF as $[a,c]$, then the middle of the interval, where the maximum of the function occurs, will be:

$$b=\frac{c+a}{2}\,.$$

In the semi-intervals $[a,b]$ and $[b,c]$ an MF can be defined using elliptical arcs. A circular shape is obtained when $b-a=c-b=1$. The modification of the curvature will modify the membership degree.

In order to simplify the treatment, and without loss of generality, we first normalize the problem to obtain circular arcs. Hence, the initial interval $[a,c]$ becomes $[a_1,c_1]$, to obtain a length of 2, $c_1-a_1=2$. Thus, $b_1-a_1=c_1-b_1=1$. If x is the coordinate before the transformation, and x_1 is the coordinate after the transformation, then the mathematical expression of the desired transformation is:

$$x_1=\frac{2}{c-a}\cdot x\,. \tag{4}$$

The coordinates a,b and c become:

$$\begin{cases} a_1=\dfrac{2a}{c-a} \\ b_1=\dfrac{2b}{c-a} \\ c_1=\dfrac{2c}{c-a} \end{cases}. \tag{5}$$

In this way, the MF that will be described below becomes a circular one. However, at the end of calculation, the inverse transformation will be applied, and the circular function will get an elliptical shape. From (4), we obtain the inverse transformation:

$$x=\frac{c-a}{2}\cdot x_1\,. \tag{6}$$

The Analytical Expression Using the Radius

Each straight line, which appears in the triangular membership function, can be bended in two directions. We obtain four possible cases, as Fig. 4 shows.

For the arcs that are in the left of b_1, the centers of circles are on the line $y_1 = -x_1 + b_1$ (Fig. 4.a and b), and for the arcs situated in the right of b_1, the centers are on the line $y_1 = x_1 - b_1$ (Fig. 4.c and d). It implies that the sign in these equations depends on the relative position of x_1 and b_1. In a single equation, the geometrical locus of centers is:

$$y_1 = \mathrm{sign}(x_1 - b_1)\cdot(x_1 - b_1), \qquad (7)$$

where $\mathrm{sign}(\cdot)$ is the signum function.

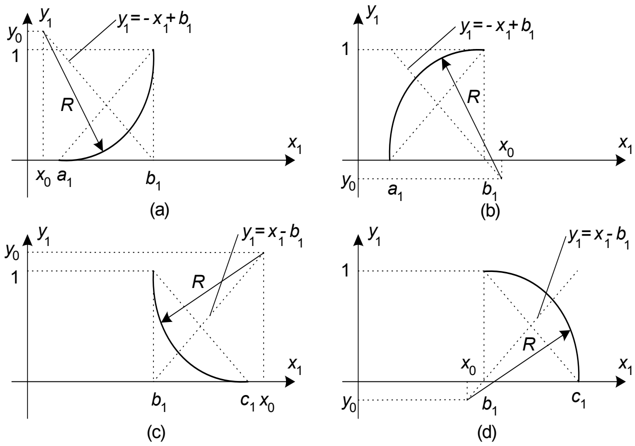

Fig. 4. The positions of the centers of circles.

From Fig. 4, the radius R, in the four cases, can be calculated following the relations [7]:

Case a) $R^2 = y_0^2 + (a_1 - x_0)^2$

Case b) $R^2 = y_0^2 + (x_0 - a_1)^2$

Case c) $R^2 = y_0^2 + (x_0 - c_1)^2$

Case d) $R^2 = y_0^2 + (c_1 - x_0)^2$.

In addition, from (7), x_0 is:

Cases a and b) $x_0 = -y_0 + b_1$

Cases c and d) $x_0 = y_0 + b_1$.

With these equations and with $b_1 - a_1 = c_1 - b_1 = 1$ the radius is:

Case a) $R^2 = y_0^2 + (y_0 - 1)^2$

Case b) $R^2 = y_0^2 + (1 - y_0)^2$

Case c) $R^2 = y_0^2 + (y_0 - 1)^2$

Case d) $R^2 = y_0^2 + (1 - y_0)^2$.

In all cases, the radius is expressed by the same equation. Solving this equation leads to [7]:

$$y_0 = \frac{1 \pm \sqrt{2R^2 - 1}}{2}.$$

Anyone can notice in Fig. 4 that $y_0 \geq 1$ for the convex arcs and $y_0 \leq 1$ for the concave arcs. Making the convention that $R > 0$ means convex arcs and $R < 0$ concave arcs, the equation from above becomes:

$$y_0 = \frac{1 + \mathrm{sign}(R)\sqrt{2R^2 - 1}}{2}. \qquad (8)$$

The other coordinate of the center (x_0), using (7) and (8), is:

$$x_0 = \mathrm{sign}(x_1 - b_1)y_0 + b_1. \qquad (9)$$

Then, the equation of the circle with radius R and center in (x_0, y_0) is:

$$R^2 = (y_1 - y_0)^2 + (x_1 - x_0)^2, a_1 \leq x_1 \leq c_1,$$

or

$$y_1 = y_0 \pm \sqrt{R^2 - (x_1 - x_0)^2}.$$

In Fig. 4.b and d, when the arcs are concave ($R < 0$ and sign "$+$" in the previous formula), the arcs are above the center. In Fig. 4.a and c, when the arcs are convex ($R > 0$ and sign "$-$" in the same formula), the arcs are under the center. The expression of y_1 can be re-written:

$$y_1 = y_0 - \mathrm{sign}(R)\sqrt{R^2 - (x_1 - x_0)^2}, a_1 \leq x_1 \leq c_1. \qquad (10)$$

The maximum bending of arcs is obtained when the arcs are tangent at horizontal or vertical lines, respectively. In this case $R = 1$ for convex arcs and $R = -1$ for concave arcs. The minimum bending is for $R \to \infty$ or $R \to -\infty$. Thus, the radius can vary in the range $(-\infty, -1] \cup [1, \infty)$. When $R \to \pm\infty$, the arc becomes a line, and the elliptical membership function becomes the triangular membership function. The graphic representations of (10), for different values of radius R, are shown in Fig. 5.

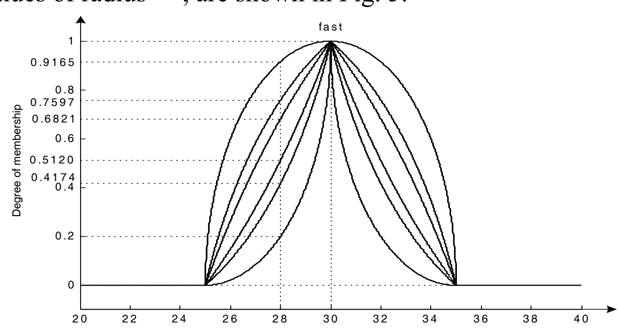

Fig. 5. The elliptical membership function.

The Analytical Expression Using the Curvature

If we want to modify the membership degree of a specific linguistic value (in our case "fast"), from the minimum value to the maximum one, we have to modify the value of radius, in (10), from 1 to $+\infty$ (from maximum bending of convex arc to straight line) and from $-\infty$ to –1 (from straight line to maximum bending of concave arc). This modification, from convex arcs or vice verse, has a discontinuity at the straight line, when a jump from $R \to +\infty$ to $R \to -\infty$ occurs. The variation may be smoothed by using the curvature r instead of the radius R. The definition of the curvature is [15]:

$$r = \frac{1}{R}. \qquad (11)$$

A straight line, which is a segment of a circle with $R \to \pm\infty$, has a curvature $r = 0$. Replacing (11) in (10), we get:

$$y_1 = y_0 - \mathrm{sign}(r)\sqrt{\frac{1}{r^2} - (x_1 - x_0)^2}, a_1 \le x_1 \le c_1. \qquad (12)$$

After calculation of y_1, the inverse transformation of the MF space is calculated with equation (6), which does the transformation $x_1 \to x$. The points y_1 are in the interval $[0,1]$, as y, so they remain unchanged: $y = y_1$.

To modify the shape of the MF using the curvature, from maximum to minimum membership degree, the curvature has to vary in the interval $[-1,1]$. However, the value $r = 0$ is forbidden. The function implemented in MATLAB accepts the minimum value for $r = \pm 10^{-10}$, meaning $R = \pm 10^{10}$, which is a very good approximation for a straight line. In this manner, the discontinuity given by the jump from $R \to -\infty$ to $R \to +\infty$ in (10) is replaced by a discontinuity given by the jump from $r = -10^{-10}$ to $r = +10^{-10}$, which is quite a small one.

GPS-INS SYNERGY ENHANCEMENT

The system pictured in Fig. 6 is used to enhance the Global Positioning System (GPS) and Inertial Navigation System (INS) synergy. From the errors point of view, the two navigation systems are complementary in short-time navigation, as well as in long-time navigation. For short-time, INS has a very good stability and GPS has a stochastic behavior. For long time, INS accumulates errors at a rate of about 1Nmi/h and GPS has a very good, stable accuracy, of about 10m.

The long time strategy is to average the output of the two systems for 30 seconds. Then, the average positions difference is calculated and is used as an input of a Fuzzy Inference System (FIS). The second input is the variation of this error since the last update (every 30 seconds). The goal of FIS is to estimate INS error for the next 30 seconds, knowing the actual error and the last error variation (Fig. 6).

For a vehicle with moderate dynamics (the position doesn't change rapidly), the variation of INS error goes up to 200m in 30 seconds. But, of course, the deterministic interval for this variable can be settled at whatever value is necessary, during the design process. The universe of discourse for "variation" is: negative (N), zero (Z) and positive (P) (Fig. 7).

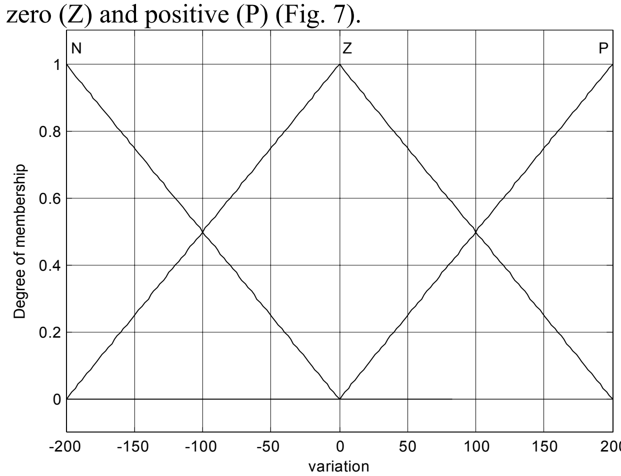

Fig. 7. The universe of discourse of fuzzy variable "variation".

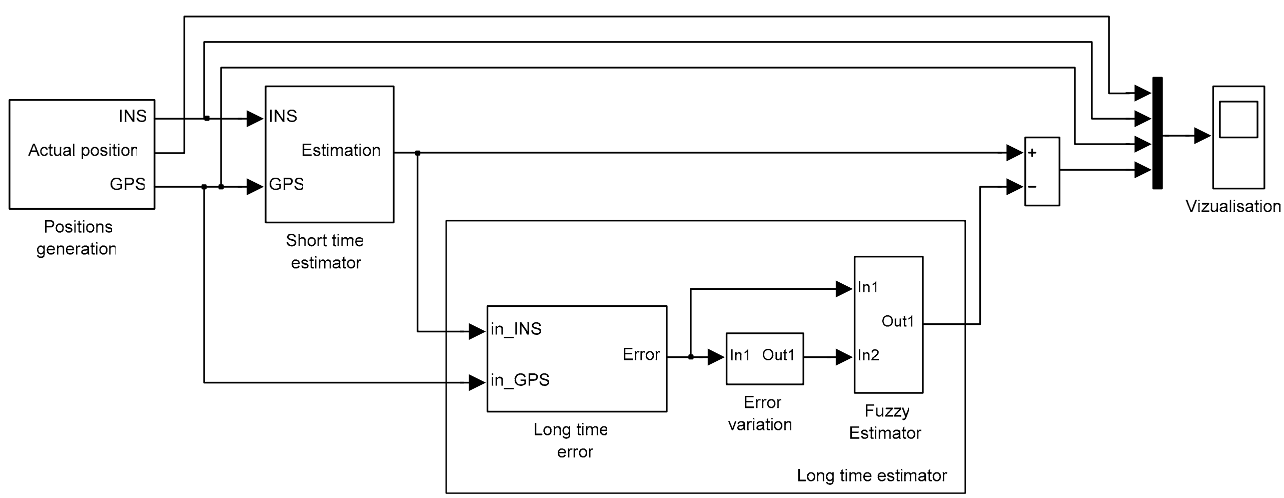

Fig. 6. The system used to enhance GPS/INS synergy.

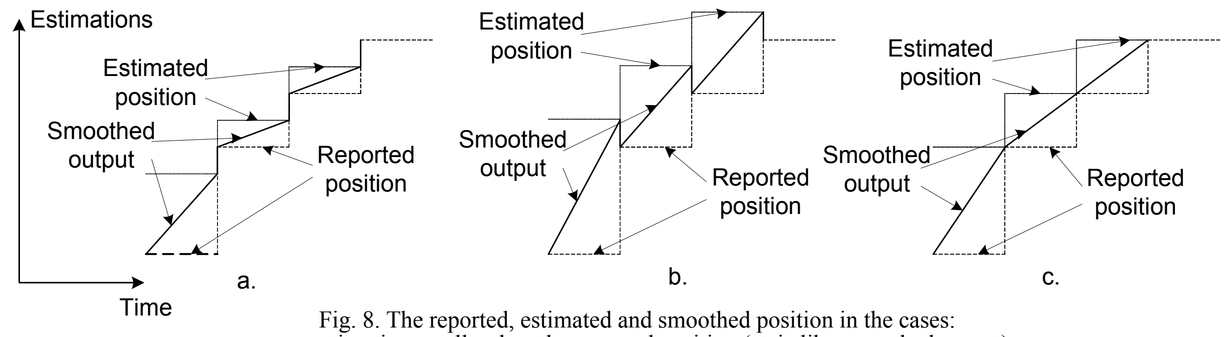

Fig. 8. The reported, estimated and smoothed position in the cases:
a. estimation smaller than the reported position (stair-like smoothed output);
b. estimation larger than the reported position (saw-toothed smoothed output);
c. ideal estimation equal to the reported position (continuous smoothed output).

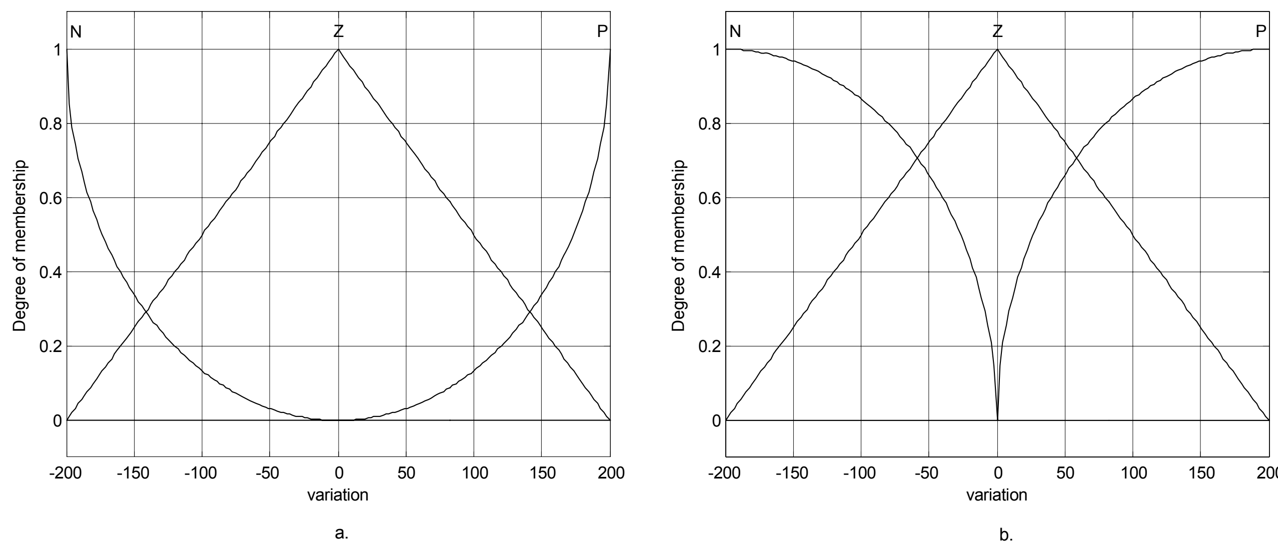

Fig. 9. The membership functions for:
a. r = 1
b. r = -1

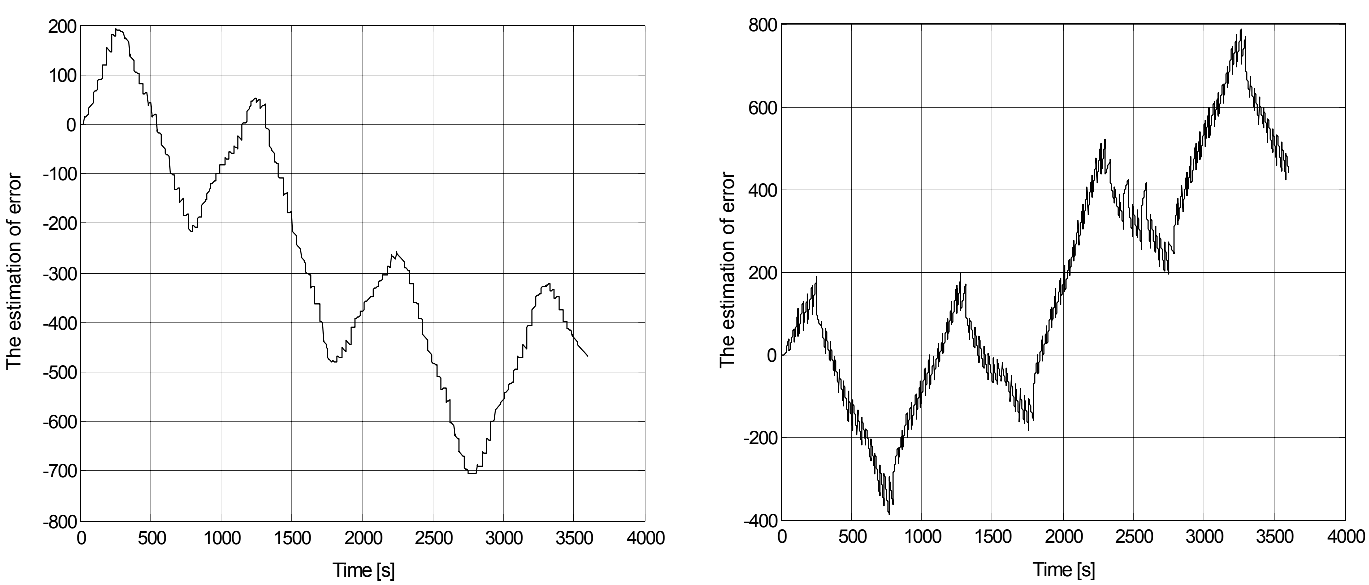

Fig. 10. The simulation results of the estimation error for:
a. - stair-like output;
b. - saw-toothed output.

To avoid jumps from an estimate to another (to smooth the output variation), the difference between the estimate (for the next 30 seconds) and the actual value is integrated. After each period, a reset signal is applied to the integrator; as result, the output value of the estimation block will change from the estimate to the actual value (Fig. 8). Three cases may occur: a) the estimate is smaller than the actual value that will occur after 30 seconds (Fig. 8.a); b) the estimate is greater than the actual value (Fig. 8.b); c) the estimate has the right value (Fig. 8.c). We have enhanced in Fig. 8 the output after integration, because of the special shape, very easy recognizable, of the three cases.

With the complete integrated navigation system, simulations were considered for different curvatures of "negative" (N) and "positive" (P) linguistic degrees of "variation". ($r=-1$ following with $r=1$ for both membership functions, see Fig. 9.a and b, respectively). The simulations results are presented in Fig. 10.a and b, respectively.

The case $r=-1$ leads to the displacement of the gravity center of the output aggregation figure to the negative and positive value. As follow, the estimations are larger than the actual values and the system response has a saw-toothed shape (Fig. 10.a). Opposite, for $r=1$, the gravity center displaces to zero and the estimations will be smaller than the actual values. This time, the shape is stair-like and is depicted in Fig. 10.b. For each case, we have also calculated the standard deviation of the error between the reported position by the two originally navigation systems and the actual position, as well as between the position reported by the hybrid system and the actual position. The results are presented in Table I.

Table I. The standard deviation of errors in rapport with the actual position

	$r=-1$	$r=1$	$r = optimum$
GPS	11.5527	11.5397	11.5638
INS	69.4914	289.7389	39.2157
HNS	25.0968	12.9651	3.6465

GPS = Global Positioning System
INS = Inertial Navigation System
HNS = Hybrid Navigation System

The manner in which the system estimates the INS error, in the optimum case, is depicted in Fig. 11, while Fig. 12 shows a zoom in the actual position, GPS position and HNS position. INS position is not presented in any figure because of the great error accumulation.

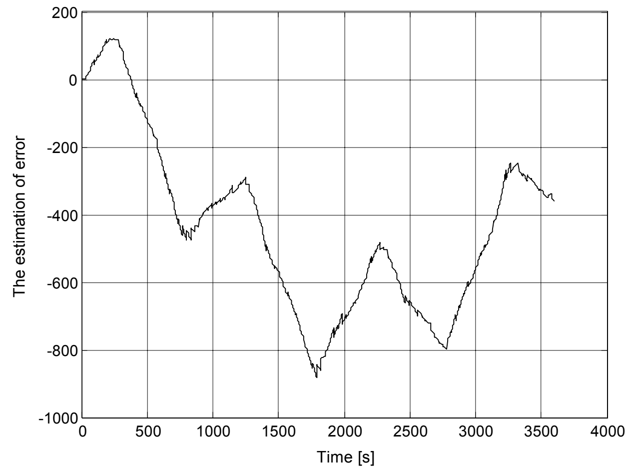

Fig. 11. The estimation error in the optimum case.

By adjusting the value of curvature in the proposed membership function, one adjusts the response of the estimator, between the extreme cases presented in Fig. 10. If the two responses have different, opposite shapes, this means that there is an optimum value, between them, for which the estimator optimally performs its task. It is intuitive the fact, if we are looking into Fig 10, that the optimum values are more in the positive part, than in the negative one (Fig 10.a is closer to the desired response than Fig. 10.b). The values used to obtain Fig. 11 are: $r=0.27$ for the negative MF and $r=0.2$ for the positive one. The zero MF remains unchanged.

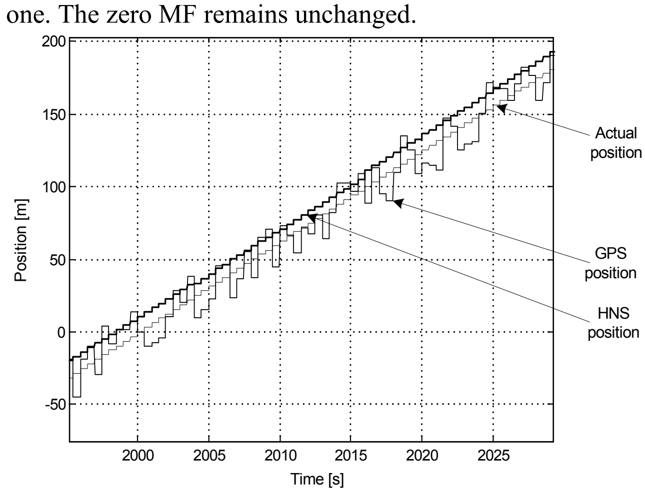

Fig. 12. Zoom into the positions reported by GPS and HNS, as well as the actual position.

As we can see in the picture above, the position reported by Hybrid Navigation System is within the GPS accuracy, without having short time random jumps.

CONCLUSION

The elliptical MF offers a good tool for Mamdani-type fuzzy inference systems. With this function, we can modify the output-input surface using only a parameter (the curvature). Moreover, with this parameter we can modify, without jumps, the overlapping of the membership functions, a vital parameter in the use of fuzzy theory in controlling dynamic systems. The paper gives an example of use in a Hybrid Navigation System. This one uses the position reported by Inertial Navigation System in short-time navigation and the position reported by Global Positioning System for long-time navigation. In the latest case, the HNS performs an estimation of the INS error during next 30 seconds. The estimator is built on fuzzy systems technology. The fuzzy subsystem for short time navigation analyzes the changes in vehicle position reported by the two navigation systems and enhances the INS solution, while the long time navigation fuzzy subsystem enhances the GPS long time accuracy.

REFERENCES

[1] D. McNeil Mayhew, "Multi-rate sensor fusion for GPS navigation using Kalman filtering", Master thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University, 1999.

[2] P.J. Escamilla-Ambrosio, N. Mort, "A hybrid Kalman filter-fuzzy logic architecture for multisensor data fusion", Proceedings of the 2001 IEEE International Symposium on Intelligent Control, September 5-7, 2001, Mexico City, Mexico, 364-369.

[3] R. Rahbari, B.W. Leach, J. Dillon, C.W. de Silva, "Adaptive tuning of a Kalman filter using the fuzzy integral for an intelligent navigation system", Proceedings of the 2002 IEEE International Symposium on Intelligent Control, Vancouver, Canada, October 27-30, 2002, 252-257.

[4] D. Loebis, R. Sutton, J. Chudley, "Review of multisensor data fusion techniques and their application to autonomous underwater vehicle navigation", Journal of Marine Engineering and Technology, No. AI, 2002, 3-10.

[5] D. Loebis, J. Chudley, R. Sutton, "A fuzzy Kalman filter optimized using a genetic algorithm for accurate navigation of an autonomous underwater vehicle", Proceedings of the 6th Conference on Maneuvering and Control of Marine Crafts, 2003.

[6] T. Abdelazim, W. Abdel-Hamid, N. El-Sheimy, E.H. Shin, "Experimental results of an adaptive fuzzy network Kalman filtering integration for low cost navigation applications", Processing NAFIPS '04, IEEE Annual Meeting of the Fuzzy Information, 2004, 844-849.

[7] A. Hiliuta, R.J. Landry, F. Gagnon, "Fuzzy corrections in a GPS/INS hybrid navigation system", IEEE Transaction on Aerospace and Electronic Systems, Vol. 40, No. 2, April 2004, 591-600.

[8] J.H. Wang, Y. Gao, "GPS-based land vehicle navigation system assisted by a low-cost gyro-free INS using neural network", The Journal of Navigation (2004), 57, 417-428.

[9] B. Kosko, "Neural networks and fuzzy systems," Prentice Hall, Englewood cliffs, 1992.

[10] L.A. Zadeh, "Fuzzy sets," *Information and control,* vol. 8, 1965, 338-353.

[11] H.-J. Zimmermann, "Fuzzy Technologien. Prinzipien, Werzeuge, Potentiale," VDI-Verlag, 1993.

[12] S.R. Swanson, "A fuzzy navigational state estimator for GPS/INS integration," Position, Location and Navigation Symposium, IEEE 1998, 541-548.

[13] H. Seranji, B. Bon, "Autonomous navigation of planetary rovers: a fuzzy logic approach," Jet Propulsion Laboratory, California Institute of Technology.

[14] B.W. Leach, R. Rahbari, J. Dillon, "Low cost strapdown IMU/DGPS integrated navigator with fuzzy logic adaptive tuning," 9th Saint Petersburg International Conference on Integrated Navigation Systems, 2002, 264-273.

[15] W. Gellert, et. al. "Kleine Enzyklopedie Mathematik," VEB Bibliographisches Institut, Leipzig, 1986.