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Validation and Performance Evaluation of Two Different Inertial Navigation System Design Approaches

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ABSTRACT

For many years, the design and implementation of a practical INS algorithm have been a subject of great interest in consideration of certain issues, such as the computing load, precision, speed/updating rate and computer memory. In the last decade, as the rapid development of computer hardware/software technology, the concern of the computing competency is not critical anymore; therefore the classic sophisticated INS design method may be simplified by taking the advantages of modern computer's capabilities.

This study is to implement and evaluate two different INS design approaches. Specifically for the classic two-speed digital INS design, the discrete INS models, the high/low speed digital integration algorithms, coning/sculling/scrolling compensations for the low speed calculation in attitude/velocity/high precision positioning determination are implemented in a C programming environment. For the simplified INS design, the single high-speed INS algorithm free of coning/sculling/scrolling compensations is investigated. Moreover by utilising Matlab Simulink's capability to solve the continuous-mode differential equations, instead of using the discrete INS models, the continuous INS models are directly employed in the simplified INS design.

The performances of the two developed INS designs are validated and evaluated inside an integrated GPS/INS solutions based on a practical loosely-coupled Kalman filter. Real-time IMU raw measurements logged from the tactic-grade Ring Laser Gyros (RLG)/accelerometers and GPS solutions corresponding to the road testing trajectory is utilised in the validation and evaluation. The comparison of the two INS solutions shows that, both of the INS designs reach to the same level of precision with respect to the provided references, however the processing time of the classic two-speed digital-mode INS is reduced 90% compared with single speed continuous-mode INS. However compared with the classic two-speed INS approach, in term of the algorithm/software complexity, it is reduced by 50-60% for the single speed INS which will permit the fast-prototyping and easy-to-understand design. Moreover a practical Kalman filter was developed by integrating INS with the GPS position and velocity data to be able to deliver the accurate and robust navigational solutions.

KEYWORDS: INS, Kalman filter, two-speed INS, IMU

1. INTRODUCTION

Inertial navigation is an autonomous process of computing position location by doubly integrating the acceleration of a point whose position is to be determined. The fundamental principle is depicted in Figure 1.

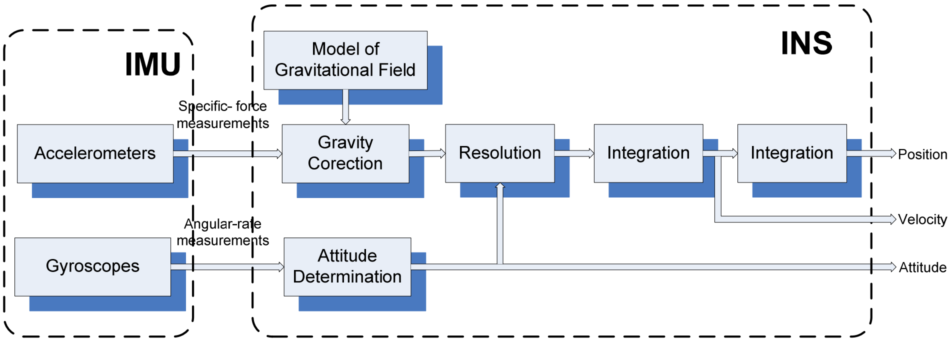


Figure 1. Basic concept of Inertial Navigation Systems

A strapdown Inertial Navigation System (INS), in which the Inertial Measurement Unit (IMU) physically mounted on the vehicle, implements the Figure 1. principle using the triad of angular rotation sensors, e.g. gyros, to determine the direction of the acceleration vector sensed by the triad of the accelerometers. By performing the integration in a digital computer, the velocity and position of the vehicle can be determined. In general, total acceleration comprises two parts: gravity acceleration and specific force acceleration. Due to the limitation of modern technology, the accelerometers only measure the specific force component of the acceleration. Therefore, to derive the total acceleration, the gravity acceleration must be calculated and added to the accelerometer measurement.

Theoretically, the attitude/velocity/position calculations are analytically described in the form of continuous differential equations that when integrated in the classical analytical continuous sense, would provide the attitude, velocity and position data. While in an actual INS, the algorithm suitable for the INS computer should be designed to achieve the same numerical results by digital integration operating at a specified repetition rate as the continuous integration of the differential equations.

Since the basic strapdown INS concept came into being, the researchers have mainly focused on the design of algorithm for the attitude integration, which is because that meeting specified angular rate sensor accuracy requirements under high dynamic angular rate inputs is generally more difficult to achieve (Savage P.G. 2000), when the strapdown angular sensors directly are exposed to the high dynamics of the vehicle. Due to the capabilities and limitations of the computer technology, the strapdown INS uses attitude algorithm based on a two-speed approach. The high speed approach with 1st order algorithm is to accurately account for the high frequency angular rate oscillation that can rectify by providing a systematic attitude build-up, the moderate speed higher order algorithm accurately accounts for larger amplitude angular motion. Taken together, the combined accuracy of two speed approach is equivalent to operation the higher order algorithm at high speed rate, however with lower computing throughput. Based on the same framework, a two speed approach can also be used for acceleration transformation/velocity integration in a dynamic environment that parallels the two speed attitude integration approach. The high speed algorithm is to account for high frequency angular and linear oscillation that can rectify into systematic velocity build-up, the low speed algorithm performs the acceleration transformation based on inputs from the high speed algorithm. For the position integration, the simple trapezoidal integration of velocity at an update rate equal to or lower than the velocity update frequency is typically employed in many modern strapdown INS computer as well in this study. In high positioning application, the two speed framework is also used to develop a high precision position integration algorithm (Savage P.G. 2000). While the intent of two speed approach was to overcome the throughput limitation of early computer technology, that limitation is rapidly becoming insignificant with continuously rapid advances in modern high speed computer technology. This provides the motives to design a simpler single speed algorithm structure in which all computations are executed at a repetition rate that is sufficiently high to accurately account for high frequency angular rate and acceleration rectification effects.

The purpose of this study is to evaluate the performance of the classic two speed INS digital integration algorithm design by comparing with a simplified single high speed INS integration algorithm design. First of all, the classic two speed INS digital integration algorithm including the coning, sculling and scrolling compensation respectively for the attitude, velocity and position calculation is implemented in C programming environment. Secondly, a simpler single high speed INS algorithm structure based on the INS analytically continuous differential equations is implemented, by utilising Matlab Simulink's capabilities to directly solve the differential equations in the continuous mode as well the high competence of modern computer technology. Finally in order to obtain a robust INS solution, a practical loosely-coupled integrated INS/GPS Kalman filter is designed to estimate the IMU sensor random noises as well to overcome the time-accumulating INS data errors by optimally coupling GPS positioning and velocity data. In the experimental results, the real-time logged tactic grade IMU raw measurements are post-processed by the two INS designs and then integrated with GPS data by the proposed Kalman filter; the road test trajectory is intentionally manoeuvred to represent different dynamics. The INS data of the two designs is compared in terms of precision, calculating time and software complexity. The results show that both of the INS designs reach to the same level of precision with respect to the provided references. Finally In the integration of INS/GPS test, the navigation solutions in terms of attitude, velocity and position are greatly enhanced by the GPS data.

2. BASIC MATHEMATICAL BACKGROUND

INS uses acceleration and angular rate measurements referenced to inertial space, but attitude, velocity and position are referenced are conventionally needed in a system referenced to the earth. Consequently the following coordinate frames defined throughout the paper are introduced to evaluate the IMU measurements (Savage P.G. 2000).

E Frame – Earth fixed frame for position location definition, Y axis parallel to the earth polar axis, X axis passes through the Greenwich meridian while parallels to the equatorial plane, Z axis is perpendicular to X and Y axis.

N Frame – Navigation frame with Z axis parallel to the upward vertical at the local earth surface, used for integrating acceleration into velocity and for defining the angular orientation.

L Frame – Locally level frame parallel to the N frame but with Z axis parallel to the downward vertical, and X, Y axes along N frame Y, X axes, used for describing the strapdown sensor frame orientation.

Geo Frame – Locally level geographic frame defined with its Z axis upward along the local geodetic vertical, Y axis north with X axis east.

B Frame – Vehicle fixed frame with axes parallel to right handed orthogonal sensor input axes. Respectively, X axis along the right wing of the vehicle, Y axis along the longitudinal axis of the vehicle and Z axis downward perpendicular to X, Y axes.

I Frame – Absolute non-rotating inertial frame used for a reference for angular rotation measurements, Y axis is directed towards the celestial north, X axis towards the vernal equinox and Z axis perpendicular to the X, Y axes.

Need to mention, all the coordinate frames X, Y, Z axes follow the right handed orthogonal rules.

3. DIGITAL STRAPDOWN INS INTEGRATION ALGORITHM IMPLEMENTATION

In an actual INS computer, as it is aforementioned, the integration functions obtaining the attitude, velocity and position are executed with digital algorithms operating at a specified repetition rate. It generally includes following steps, firstly integration of INS angular rate sensor data into attitude – Attitude Integration, secondly using attitude data to transform INS accelerometer data from B frame in N frame – Velocity Integration and finally integration of N frame velocity into position – Position Integration.

As an example of the two speed digital integration framework implementing Attitude, Velocity and Position Integration, the two speed algorithm of Attitude Integration is explained in details in Figure 2. $C_{B(m)}^{L(n)}$ is the attitude matrix describing the relation between B Frame to

L Frame to be determined. A simple high speed 1st order algorithm accounts for small amplitude high frequency angular oscillations that can rectify into systematic attitude build-up in terms of Coning compensation $\underline{\beta}_m$ in l loop (1 - 4 KHz). Depending on nowadays inertial sensor technology, other than directly providing the angular rate, most of angular rate sensor output the integrated angular rate increments $\Delta\underline{\alpha}_l$ over a certain sampling period. By taking the outputs from high speed l loop, a moderate speed higher order algorithm – accurately account for larger amplitude angular motion over the moderate speed update cycle period – in *m* loop (50 – 200 Hz). B Frame attitude change from epoch m to epoch m-1 $C_{B(m)}^{B(m-1)}$ is achieved by solving the rotation vector differential equation $\dot{\phi}_m$ in digital approximation $\underline{\phi}_m$.

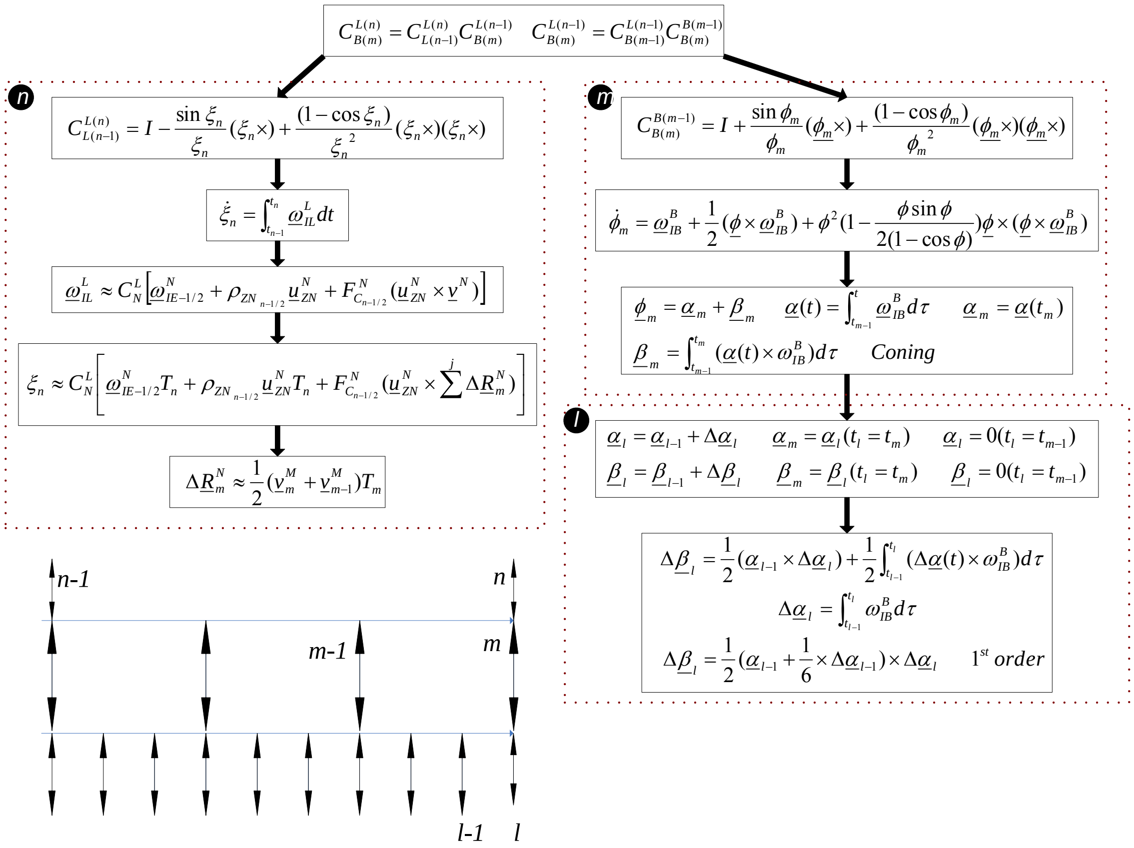


Figure 2. Scheme of Attitude Calculation

To accomplish the attitude calculation depicted in Figure 2, L Frame attitude change caused by the vehicle translational motion needs to be updated. Due to small changes in the velocity and position over the calculation interval, the L Frame rotation vector ξ_n is very small. Hence it can be calculated by approximating a non-rotation L Frame rate vector $\underline{\omega}_{IL}^L$ in the low speed update cycle period – in *n* loop (50 Hz).

Based on the same two speed framework, the velocity integration is depicted in the Figure 3. B Frame attitude change from epoch m to epoch m-1 $C_{B(m)}^{B(m-1)}$ is used to determine the direction the acceleration vector. Consequently the calculation of $C_{B(m)}^{B(m-1)}$ containing the l loop rate Coning compensation is included in the velocity integration. When coupling with attitude

Coning effect, the acceleration vector generates linear velocity oscillation. Therefore a high speed 1st order algorithm accounts for high frequency angular/linear oscillations that can rectify into systematic velocity build-up – Sculling compensation in *l* loop (1 - 4 KHz). A moderate speed higher order algorithm performs the acceleration transformation based on inputs from the high speed algorithm – in *m* loop (50 – 200 Hz). Meanwhile in *m* loop, the velocity caused by the gravity and Coriolis acceleration should be calculated. Because gravity and Coriolis acceleration is a function of the vehicle position and earth rotation/translational rate respectively, it can be calculated by approximating a non-rotation L Frame rate vector $\underline{\omega}_{IL}^{L}$ in the low speed update cycle period – in *n* loop (50 Hz).

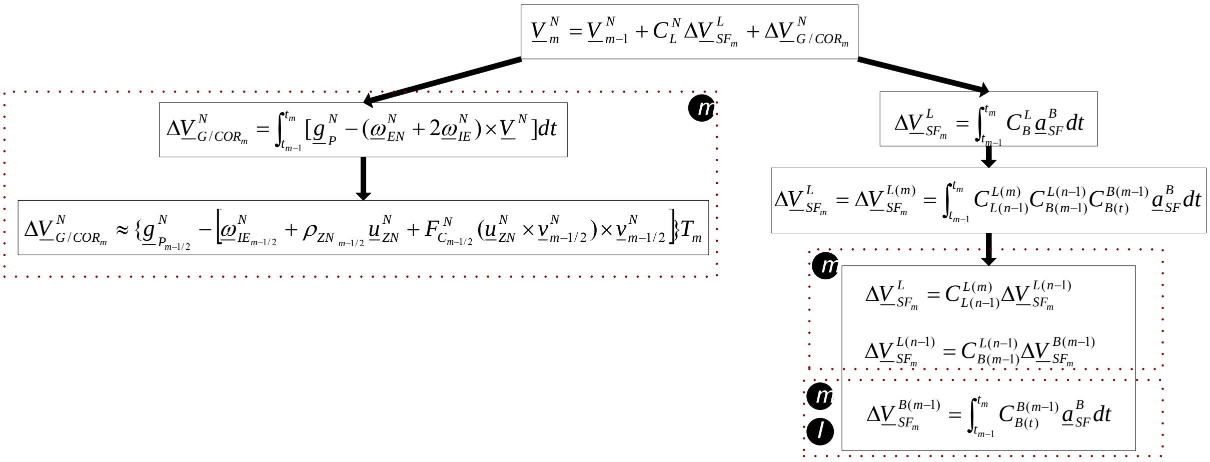


Figure 3. Velocity Integration

In the position determination, the simple trapezoidal integration as well the high resolution position algorithm containing high speed Scrolling compensation is implemented to account for the dynamic attitude and velocity changes within the position update period.

4. CONTINUOUS STRAPDOWN INS INTEGRATION ALGORITHM IMPLEMENTATION

The same basis as the digital INS algorithm, theoretically, the attitude/velocity/position calculations are analytically described in the form of continuous differential equations that when integrated in the classical analytical continuous sense, would provide the attitude, velocity and position data. In this study, by utilising Matlab Simulink's capabilities to directly solve the differential equations in continuous mode and the high competence of modern computer technology, a simpler single high speed INS algorithm structure based on the INS analytically continuous time-rate differential equations is implemented as well. The attitude is achieved by the integration of the attitude time rate equation given by:

$$\dot{C}_B^L = C_B^L (\omega_{IB}^B \times) - (\omega_{IL}^L \times) C_B^L, \quad \omega_{IL}^L = C_N^L (\omega_{IE}^N + \omega_{EN}^N) \tag{1}$$

Where, C_B^L Attitude matrix, $\omega_{IB}^B \times$ skew-symmetric matrix of the angular rate vector in B frame, $\omega_{IL}^L \times$ skew-symmetric matrix of the angular rate vector caused by the translational motion in L frame, ω_{EN}^N the angular rate of N Frame relative to E Frame, i.e. so-called transport rate, ω_{IE}^N earth rotation rate. The Matlab implementation of attitude integration is given by Figure 4.

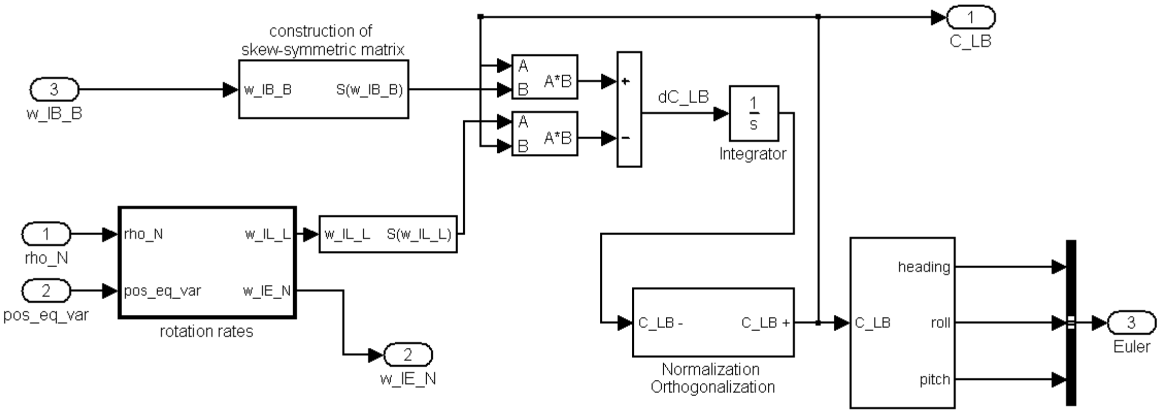


Figure 4. Attitude Integration

The velocity is determined by the integration of the velocity time rate equation given by:

$$\dot{v}^N = a_{SF}^N + g_P^N - (\omega_{EN}^N + 2\omega_{IE}^N) \times v^N \ , \ g_P = g - \omega_{IE} \times (\omega_{IE} \times R) \tag{2}$$

Where, v^N is velocity vector, a_{SF}^N specific force vector, g_P Plumb-bob gravity, g is Standard gravity, R is position location vector from the earth center. The Matlab Simulink implementations of velocity integration and transport rate are given in Figure 5 and 6 respectively.

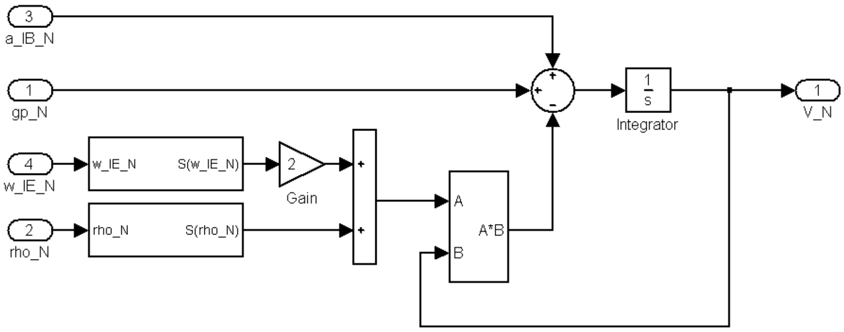


Figure 5. Velocity Integration

The position determination can be accomplished by integrating the follow equation:

$$\dot{C}_N^E = C_N^E(\omega_{EN}^N \times) \qquad \dot{h} = v_{\perp}^N \tag{3}$$

Where, $\omega_{EN}^N \times$ the skew symmetric matrix of transport rate, C_N^E Direction Cosine Matrix (DCM) from N frame to E frame, i.e. the so-called position matrix, $v_{\perp}^N$ vertical component of velocity vector.

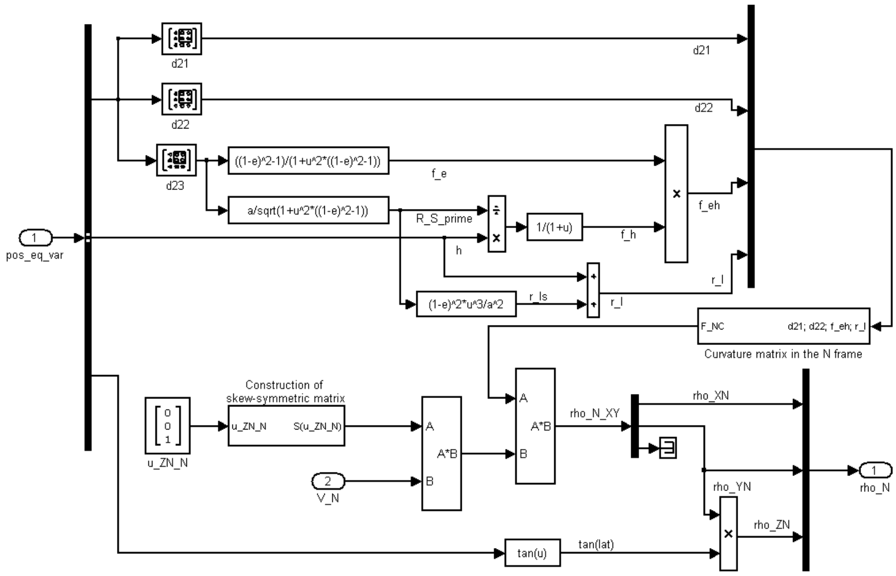


Figure 5. Implementation of Transport Rate

5. INTEGRATED KALMAN FILTER IMPLEMENTATION

INS stand-alone solution has limited by its time-growing errors due to the imperfection of inertial sensors, i.e. inertial sensor bias, random noise, scale factor instability etc. In the comparison with INS, Global Navigation Satellite System (GNSS), such as GPS, GLONASS and Galileo, is capable of delivering position and velocity information ascertained with time-independent precision. While because GNSS is vulnerable to the environment interference, such as radio emission, building blockage of the signal etc, therefore as a totally autonomous navigation solution, INS is commonly coupled with GNSS by Kalman filter technique to augment the over-all performance.

Generally the design of Kalman filter error state transition matrix is critical to the integration system performance. The transition matrix designed in this study is on the basis of INS dynamic error models which are given by (Savage P.G.):

$$\begin{aligned}
\dot{\underline{\Psi}}^N &= -C_B^N \delta\underline{\omega}_{IB}^B - \underline{\omega}_{IN}^N \times \Psi^N \\
\delta\dot{\underline{V}}^N &= C_B^N \delta\underline{a}_{SF}^B - \underline{a}_{SF}^N \times \Psi^N + \delta\underline{g}_{Mdl}^N - (2\underline{\omega}_{IE}^N + \underline{\omega}_{EN}^N) \times \delta\underline{V}^N \\
\delta\dot{\underline{R}}^N &= \delta\underline{V}^N - \underline{\omega}_{EN}^N \times \delta\underline{R}^N
\end{aligned} \tag{4}$$

Where, $\underline{\Psi}, \delta\underline{V}, \delta\underline{R}$ Attitude, velocity and position error parameters, C_B^N DCM from B to N frame, $\delta\underline{g}_{Mdl}^N$ Plump-bob gravity error, $\underline{\omega}_{IN}^N$ Navigation frame rotation rate with respect to the inertial frame, by definition which equals to $\underline{\omega}_{IE}^N + \underline{\omega}_{EN}^N$, the definitions of $\delta\underline{\omega}_{IB}^B$ are same to the Eqa. (1)-(4). In some documentation, the equations (3) refer to INS ψ-angle error model, in which the INS error parameters, i.e. attitude/velocity/position errors are defined in the E Frame and expressed in N Frame.

The Kalman filter error state transition matrix can be achieved by discretizing the time-rate differential Eqa. (4) with the proper algorithm, in this study 2nd order algorithm is utilised:

$$\Phi_m = I + \Delta\Phi_{\lambda\lambda_m} + \frac{1}{2}\Delta\Phi_{\lambda\lambda_m}{}^2 \qquad \Delta\Phi_{\lambda\lambda_m} = \int_{t_{m-1}}^{t_m} A(t)dt = \begin{cases} A(t_{m-1})\cdot T_m \\ \frac{1}{2}[A(t_{m-1}) + A(t_m)]\cdot T_m \end{cases} \qquad T_m = t_m - t_{m-1} \tag{5}$$

Where, *A(t)* is the continuous-mode INS dynamic error model represented by Eqa(4), *m* is the time interval being used to transit the INS error states between two epochs, e.g. in this study, it is the same as the *m* cycle update rate, i.e. 100Hz. Because the time interval T_m is small, *A(t)* paramters can be treated as constants or slowly changing variables in the integration during the T_m interval. For instance, the Matlab Simulink implementation of INS dynamic error model and the discretization is given by Figure 6.

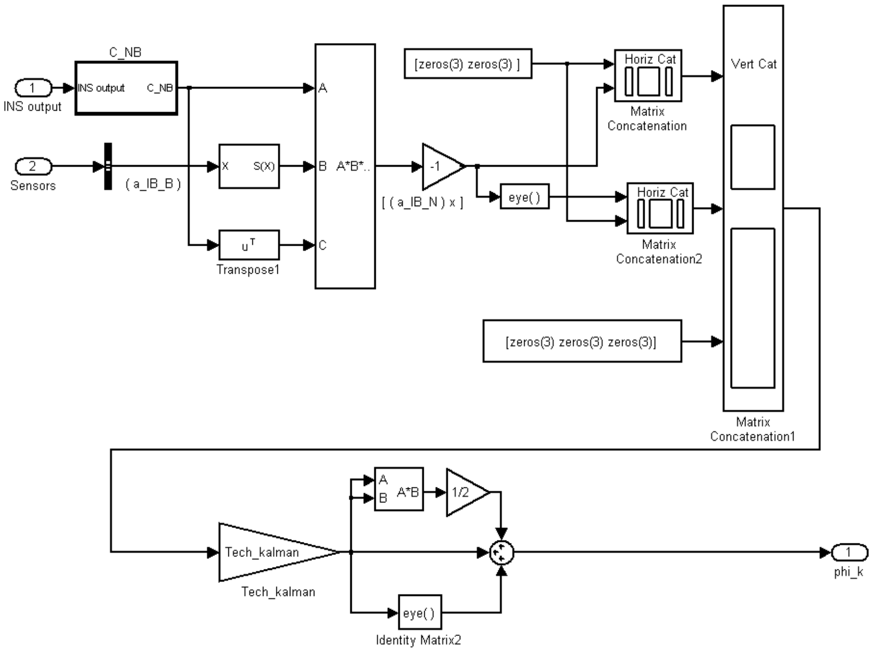


Figure 6. Simulink implementation of Kalman filter error state transition matrix

6. SYSTEM VALIDATION AND PERFORMANCE EVALUATION

The road test was conducted in a parking lot in Laval, Québec on the 24th July 2007. In the experimental results, the real-time logged tactic grade IMU raw measurements are post-processed by the two INS designs and then integrated with GPS data by the proposed Kalman filter; the road test trajectory is intentionally manoeuvred to represent different dynamics. In the test, IMU Equipment "Honeywell 1700 AG58" in Figure 7(a) was utilised, which specifications are provided by Table 1. The GPS receiver used in the test is NavAtel DL-4 in Figure 7(b). The test assembly is depicted in Figure 7(c)/(d). The NavAtel integrated GPS/INS solutions "Best INS PVA" were employed in the test as the reference.

Table 1. Specifications of Honeywell 1700 AG58

Gyro				Accelerometer			
Input Range (deg/sec)	Rate Bias (deg/hr)	Rate Scale Factor	Angular Random Walk (deg/rt-hr)	Input Range (g)	Bias (mg)	Scale Factor	Acceleration Random Walk (ug/rt-Hz)

1000	1	1.0/ (8589934592.0)	0.125	50	1	0.3048/ (134217728.0)	34

In the test, first of all the stand-alone INS solutions respectively from C and Simulink implementations are compared with the reference. The stand-alone INS solutions are obtained by processing the raw IMU data, i.e. triad specific forces/angular rates. Figure 8-10 (a) depicts the INS stand-alone solutions from Simulink approach. The test is for 300s, the processing time is about 60s. By processing the same raw IMU data, the INS stand-alone solutions by C approach are obtained and depicted in Figure 8-10 (b). The processing time by C INS software is about 4-5s. Comparing the processing time, obviously C approach processes much faster than Simulink, which makes C program is ideally for a real time INS solution than Simulink.

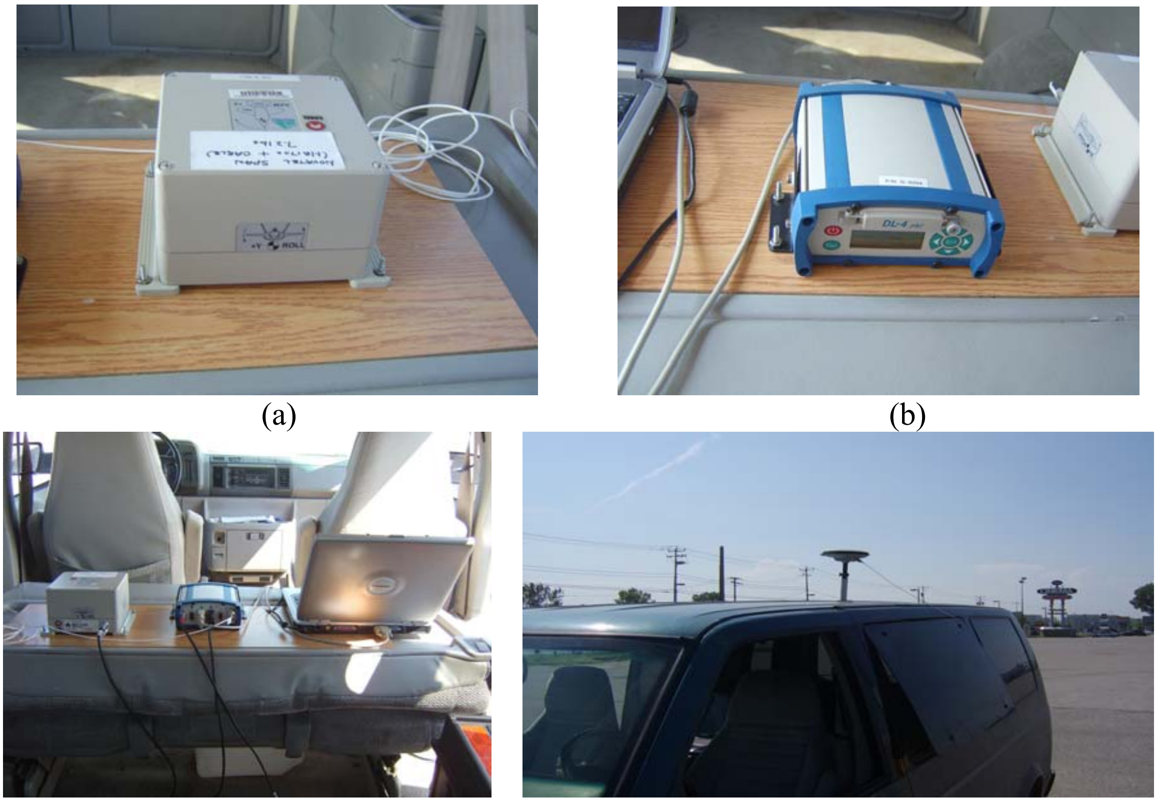
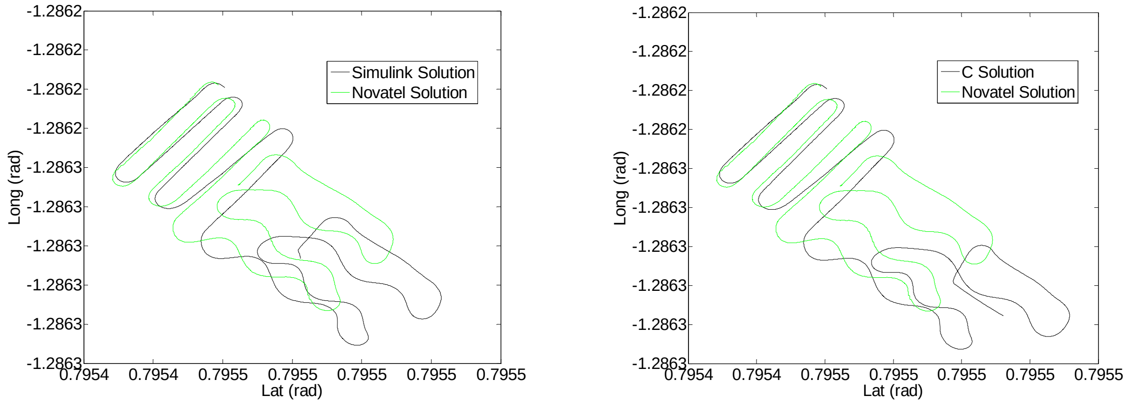

(a) (b)
(c) (d)

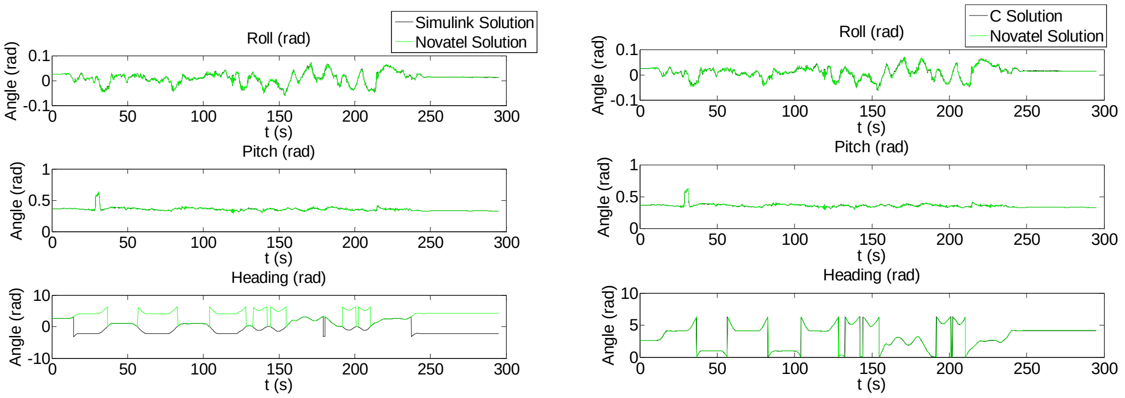
Figure 7. Experimental test equipments and setup

By comparing with NovAtel best PVA, depicted in Figure 9-11, the stand-alone INS attitude, velocity and position solution from C/Simulink approach reach the same level precision. It can be found clearly that the stand-alone INS has the time-dependent errors, which are mainly caused by the IMU sensor bias and noise. It also can be concluded that both of the Simulink and C INS approaches deliver the stand-alone INS solutions with the same accuracy, while C program is much faster than Simulink in term of processing time.



(a) (b)

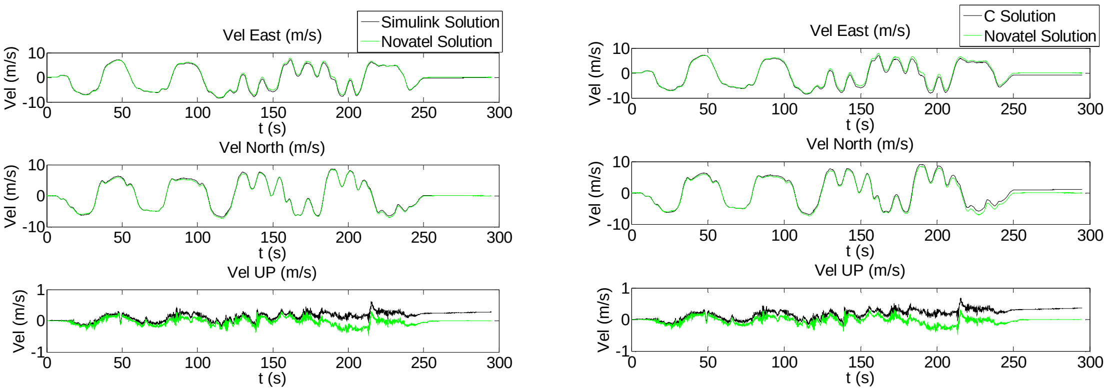
Figure 8. INS stand-alone solution from Simulink and C vs. NovAtel Best Position solution



(a) (b)

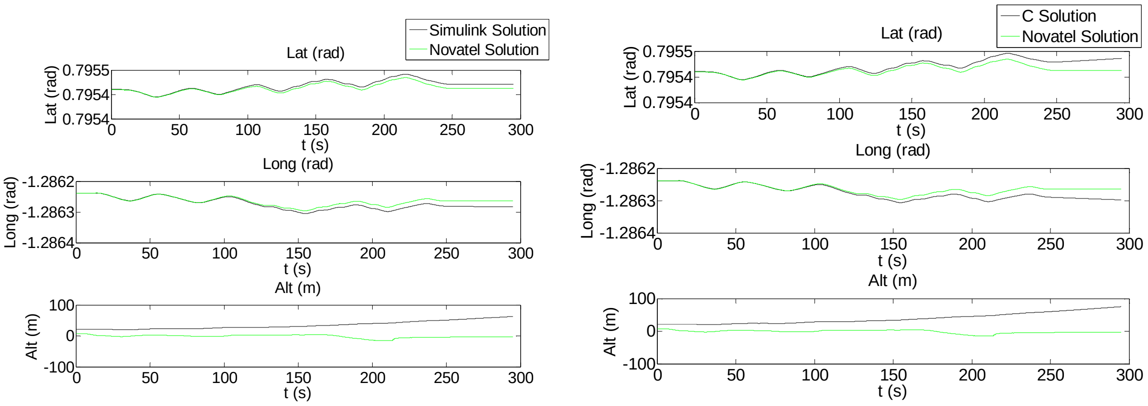
Figure 9. INS stand-alone solution from Simulink and C vs. NovAtel Best Attitude solution

Need to mention that in Figure 9 (a), the big angle heading difference between Simulink solution and the reference is 2π.



(a) (b)

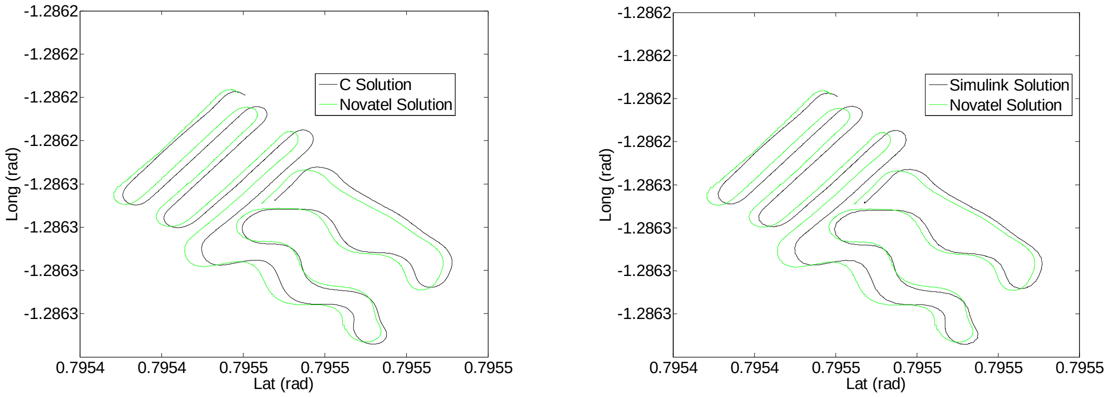
Figure 10. INS stand-alone solution from Simulink and C vs. NovAtel Best Velocity solution



(a) (b)

Figure 11. INS stand-alone solution from Simulink and C vs. NovAtel Best Position solution

Due to the IMU sensor errors, the INS stand-alone solution has the time-growing errors. To estimate and compensate those errors, referring to the aforementioned reasons, the Kalman filter-based INS/GPS integration is the ideal solution to deliver the optimal attitude, velocity and position. Moreover according to the basic principles of INS, the vertical channel is naturally divergent in the INS stand-alone calculation, which can be proved by Figure 11(a)/(b) – refer to altitude solutions. To make the altitude calculation converge, the external correction for the altitude control is required. By utilising the accurate GPS altitude information, the altitude control of INS can also be achieved by the integration of INS/GPS. Figure 12 depicts the integrated INS/GPS solution compared with the reference.



(a) (b)

Figure 12. INS/GPS integration solution from Simulink/C vs. NovAtel Best Position solution

In the test, according to the reference, the distance between each straight-line trajectory is about 3 meters. Therefore the errors between integrated INS/GPS solution and the reference is roughly about 0.5 meter, which can be seen from Figure 12.

Figure 13-15 show the errors of Simulink/C integration solution compared with NovAtel Best PVA. From the results, it can be observed that the C approach based INS/GPS integration has smaller and less noisy errors than Simulink-based INS/GPS integration solutions, which is because the digital integration algorithm used in C INS/GPS integration are more accurate and well designed specified for the INS calculation than the simplified continuous-mode integration algorithm employed in Simulink INS/GPS algorithm.

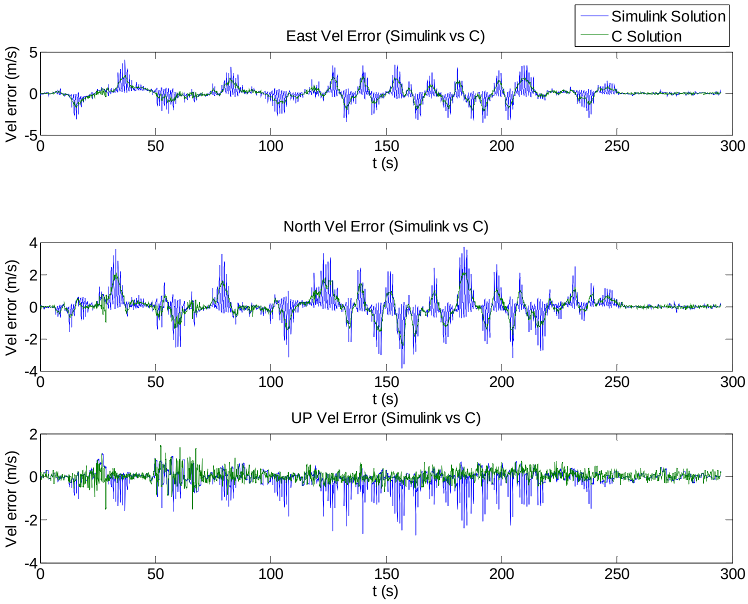


Figure 13. INS/GPS integration velocity errors of Simulink/C compared with NovAtel Best Velocity solution

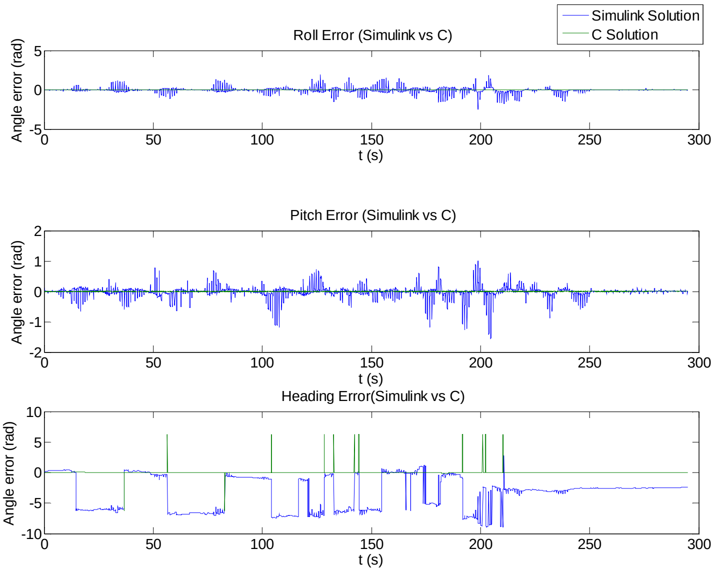


Figure 14. INS/GPS integration velocity errors of Simulink/C compared with NovAtel Best Attitude solution

It is the same as in Figure 9, the big heading errors is 2 π adjustment, where2 π and 0 indicate the same heading, i.e. true north in Figure 14.

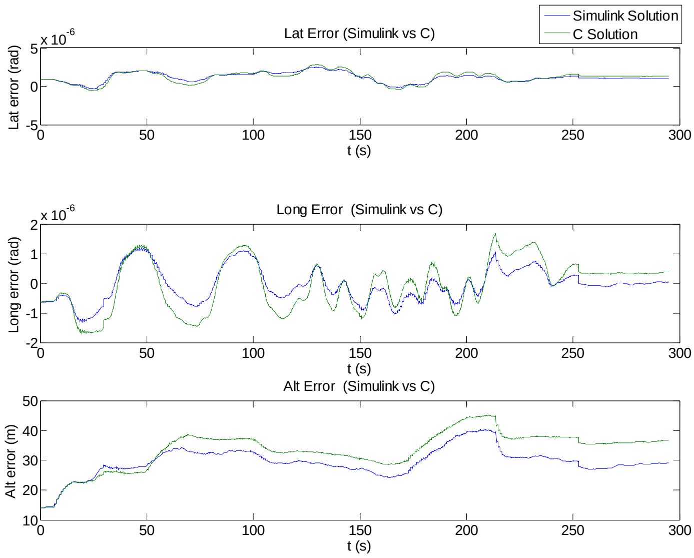


Figure 15. INS/GPS integration velocity errors of Simulink/C compared with NovAtel Best Position solution

6. CONCLUSION

This paper focuses on the design of the INS algorithm. Two INS approaches respectively in C programming-based digital mode and Simulink-based continuous mode are implemented. Real data road test validates the both proposed designs. Firstly in the stand-alone INS test, with fairly big IMU sensor systematic bias and noises, however the stand-alone INS solutions from C and Simulink approaches follow the reference trajectory. This validates the stand-alone INS solutions by C/Simulink implementation. With the comparison of C and Simulink INS, both of the approaches reach the same level of precision, nevertheless the C programming-based stand-alone INS is improved 90% in term of the processing time consumption compared with Simulink-based one

Moreover the followed validation test shows that Kalman filter is able to effectively estimate/compensate the raw IMU random noises by utilising the aid of the GPS position and velocity data. The integrated INS/GPS delivers the accurate solution compared to the reference. According to the results, it can be concluded that the INS solution can be improved by mitigating the inherent IMU sensor errors with the aid of integrating the GPS data.

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