

# A Robust RTK Software for High-Precision GPS Positioning

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## BIOGRAPHIES

Thomas Delaporte is a Master Engineering student at the École de Technologie Supérieure (ETS), Montréal, Canada. He has finished his engineering degree in 2007 at Institut Supérieur d'Electronique Numérique (ISEN) in Lille, France, where he specialized in digital electronic and telecommunications. Since 2007, he is involved in the HRNAV project of the Navigation Research Group (NRG) of LACIME laboratory. His master degree topic concerns the improvement of the precision robustness of real-time kinematics DGPS for airborne survey.

After the completion of a M.Sc. in Satellite Communication Engineering at the University of Surrey in 1993 (Guildford, UK), René Jr. Landry received a PhD degree at SupAéro / Paul-Sabatier University and a Post Doc in Space Science at the National French Space Industry (CNES), both at Toulouse in France (1997 and 1998 respectively). Since 1999, Professor Landry joins the department of electrical engineering at ETS where his major interest concerns the increase of robustness and precision in navigation. He is actually working on several digital signal processing applications and new innovative architectures for GNSS receiver design, anti-jamming and indoor navigation technologies, sensors integration and Inertial Navigation Systems (INS).

Dr. Mohamed Sahmoudi received a PhD degree in signal processing and communications systems from Paris-Sud (Paris XI) University in collaboration with Telecom Paris in 2004, and a M.S. degree in probability and statistics from Pierre and Marie Curie (Paris VI) University in 2000. During his PhD, he was an assistant lecturer at the Electrical Engineering Department of Ecole Polytechnique, France, then a lecturer in statistics at Paris-Dauphine University. From 2005 to 2007, he was a post-doctoral researcher working on GPS and signal processing at the center for advanced communications of Villanova University, PA, USA. Since Août 2007, he joined the LACIME laboratory of the ETS of Montreal to lead a team of research on Real-Time Kinematic for robust and precise positioning. He is also involved in other projects including interference and multipath mitigation, advanced signal processing for weak GNSS signal acquisition and tracking, and software developments.

Jean-Christophe Guay is a Master Engineering student in the department of electrical engineering at ETS

(Montreal, Canada). He has received an electrical engineering degree at the same University in 2007. As part of his master degree project, he has already integrated the WAAS capabilities in the LACIME GNSS receiver. His major interest concerns real time digital signal processing, software defined radio (SDR) and the development of a SDR-based GNSS receiver for severe environment.

## ABSTRACT

In this paper, we consider an airborne positioning application in which we need to achieve the highest precision of a real-time kinematic (RTK) GNSS system for surveying purposes in Canada. The challenges for such real-time applications are related to the long baseline (more than 100 km), introducing different kind of important errors including high dynamics, potential cycle-slips (i.e. sudden jumps in the carrier phase signal), ionosphere model degradations, multipath and interferences. Resolving each of these problems has been a cornerstone in the RTK positioning state-of-the art. Therefore, we intend to improve the RTK performance in severe environments.

This paper introduces a new procedure to improve the overall RTK robustness by incorporating some effective robustness techniques in the Kalman filter-based RTK approach. More precisely, we propose a cycle-slip detection and correction procedure using a robust statistical test based on the innovation processes of the Kalman filter. When a cycle slip is detected, the concerned ambiguity resolution is re-initialized and the robust M-estimate of the innovation covariance is used to robustify the adaptive RTK algorithm.

The proposed scheme combines the resulting robust Kalman filter version and the LAMBDA method for ambiguity resolution. The main idea is to perform an optimal detection of outliers and wrong estimations values when some of the data observations are missing or corrupted. Thus, the developed procedure permits to remove, at the same time, the contributions from non-Gaussian interferences and undesired measurements. Real-time and post-processing test results show the effectiveness of the developed RTK software.

## 1. INTRODUCTION

Recently, the precise positioning with GPS at the centimeter-level accuracy has attracted much attention due to its broadened applications in navigation and surveying, intelligent transportation systems, guidance and control of aircraft, rescue operations and military needs [4]. This real-time kinematic (RTK) GNSS technology uses carrier phase measurements from two GNSS receivers. Because of the Doppler shift induced by the relative motion of the satellite and the receiver, the carrier phase is changing in frequency. So the received carrier phase is related to the phase of the carrier at the satellite through the signal time of propagation from the satellite to the receiver. Thus, the carrier-phase observable would be the total number of carrier cycles plus a fractional cycle part between the satellite and the receiver. Unfortunately, there is no way to know the exact number of the entire cycles, introducing an integer number of ambiguity in the carrier tracking process. To use the carrier phase measurements for precise positioning, this unknown number of cycles must be estimated along with the other unknowns of interest that may include the receiver's coordinates, and the velocity of the user's movement.

The problem of precise relative positioning with carrier phase turns out to be a problem of correct estimation of integer ambiguities. In the literature, there are many approaches for the RTK solution including essentially the LAMBDA method [3], and the Kalman filter algorithm with different implementations [1], [5], [6]. However, as the distance between the rover (i.e. rover receiver) and the reference (i.e. base receiver with known location) increases, the problem of ambiguity resolution becomes more challenging due to the spatial decorrelation of differential atmospheric biases and orbital errors. Furthermore, when the pseudorange code-based measurements are incorporated with the carrier-phase observations, multipath can be the dominated error. Consequently, some jumps and biases in the positioning results may appear due to the wrong estimation and/or to the cycle slips and corrupted observations. Thus, the mismodeling effects must be taken into account and the outliers and undesired data contributions must be efficiently removed. Therefore, we believe that if an efficient consideration of the error's model is taken into account, the user can also use RTK for long-range baseline (>100 km). The use of such system will be a cornerstone for the development and success of airborne survey. In this paper, we introduce a new strategy for cycle slips detection and correction, based on a robust technique of signal jumps processing. The main idea consists of using the robust covariance matrix of the innovation process of the Kalman filter to detect the outliers and non-regular behaviors in the Kalman filter estimation process. The robust estimation method has been used in the RTK literature as a robust approach for data processing [7], [15]. However, in this contribution we take further the advantage of this powerful method to measure the effectiveness of the ambiguity and navigation coordinates resolution. Moreover, based on the above detection stage, we suggest some actions to be

considered to achieve robust ambiguity correction and rover position estimation in the case of corrupted observations. In that case, we propose to reinitialize the ambiguity search process and to use the developed robust version of the navigation filtering algorithm.

For convenience, we present the developed RTK-GNSS software that has been developed at the LACIME laboratory of ETS University for robust and precise long-baseline navigation. This software is used to provide robust ambiguity resolution and solve the navigation equations in post-processing as well as in real-time. The graphic interface monitoring of this software is a friendly graphic user interface (GUI), and it provides also the capability of full parameterization and configuration of the algorithm in real-time for analysis purposes.

## 2. PRINCIPLE OF RELATIVE PRECISE POSITIONING.

The RTK algorithm needs the carrier phase measurements or basically the Accumulated Doppler Range (ADR) to obtain a centimetre-level of precision. The precision of carrier phase measurements is typically 0.025 cycles (3.5 mm), which make it a powerful system to obtain even millimetre precision [4], [8]. The pseudo-range measurements are noisier and can not achieve such centimetre accuracy, but the carrier phase ambiguity resolution can be a challenging problem to reach that goal. Indeed, the main problem with the ADR measurements is its ambiguity in terms of the number of total carrier phase cycles between the rover and the GPS satellites (i.e. the integer number of cycles is unknown).

The pseudo-range and the ADR are expressed as below:

$$\rho = r + I + T + c \cdot (\delta t_r - \delta t_b) + \varepsilon_p \quad (1)$$

$$\lambda\phi = (r - I + T) + c \cdot (\delta t_r - \delta t_b) + \lambda N + \varepsilon_\phi, \quad (2)$$

where:

$\rho$  is the pseudo range measurement,  
 $\phi$  is the carrier phase measurement (ADR) (cycles),  
 $I$  and  $T$  are the ionospheric and tropospheric delay,  
 $\delta t_r$  and  $\delta t_b$  are the clock biases of the rover and satellite respectively,  
 $c$  is the speed of light,  
 $\lambda$  is the wavelength of the frequency L1 or L2,  
 $N$  is the ambiguity of the carrier phase,  
and  $\varepsilon_p$  and  $\varepsilon_\phi$  are the residual errors.

For each satellite, we use two set of measurements according to equations (1) and (2) for the frequencies  $L_1$  and  $L_2$ . The same kind of code and carrier phase observations is used from the base receiver and from the rover receiver.

The principle of precise positioning consists of using double difference (DD) of measurements between the

rover and the base receivers. By performing the first difference, the common errors of satellites, tropospheric, ionospheric delays, as well as, ephemeris, and satellites clock delays and drifts are removed from the estimation problem. By differencing two single differenced measurements, for example the single difference measurements of satellite k from the satellite l, the clock bias and the receiver clock bias and the receiver clock drift are removed from the estimation problem. All satellite's signals are differenced toward the same reference satellite k. The remaining errors are mostly the multipath, the satellite orbit errors and the bias of the center of antennas [8].

Thus, we obtain the following double differences of the ADR on the frequency L1 or L2:

$$DD_\phi = \lambda\phi_{rb}^{(kl)} = \lambda(\phi_r^{(k)} - \phi_b^{(k)}) - \lambda(\phi_r^{(l)} - \phi_b^{(l)}) \quad (3)$$

$$\lambda\phi_{rb}^{(kl)} = r_{rb}^{(kl)} + \lambda N_{rb}^{(kl)} + \mathcal{E}_{\phi,rb}^{(kl)}, \quad (4)$$

where  $\phi_r^{(k)}$  and  $\phi_b^{(k)}$  stand for the carrier phase of the satellite k received by the rover and the base, respectively. The double difference rover-base between satellite k and satellite l is denoted by  $\phi_{rb}^{(kl)}$ . The same principle can be applied to the Doppler frequency and to the pseudo-range measurements. Then, the search of the ambiguities has been transformed to a search of the double difference ambiguities (DDA).

In non-ideal environments, such as in presence of multipath, jamming signals, spoofing and non-Gaussian noise, the measurements statistics need to be robustly estimated. Indeed, the direct statistics computation from input data may not represent the required actual GNSS information. Therefore, we propose in this contribution to include a statistical test step in the filtering algorithm in order to obtain a much accurate relative state estimation. For convenience of the paper presentation, we summarize first the formulation of the extended Kalman filter estimation method.

### 3. KALMAN FILTER FOR FLOAT AMBIGUITIES ESTIMATION

#### 3.1 Kalman filter –based RTK algorithm

We use a classic Kalman filter with a state space vector composed of the rover-base baseline, the DDA carrier phase vectors for L1 and L2 and the velocity vector of the rover. The model is derived from the constant velocity model, as it is well documented in the literature.

For the two frequencies L1 and L2, we express the state vector as,

$$X = [x \quad y \quad z \quad \lambda_1 N_{rb}^{(kl)} \quad \lambda_2 N_{rb}^{(kl)} \quad dx \quad dy \quad dz]^T. \quad (5)$$

The size of the DDA vector  $N_{rb}^{(kl)}$ , corresponds to the number of used satellites and differenced with the reference satellite k.

The measurement vector DD is composed of the double difference pseudo-range measurements, carrier phase measurements and Doppler frequency, and this for the two frequencies L1 and L2.

$$DD = [DD_\rho^{L1} \quad DD_\rho^{L2} \quad DD_\phi^{L1} \quad DD_\phi^{L2} \quad DD_d^{L1} \quad DD_d^{L2}]^T. \quad (6)$$

The Kalman filter needs an observation model to link the observation vector to the state space vector. It is represented by a matrix H. The observation model for the double difference carrier phase measurements is expressed as follows:

$$DD_{\phi L1} = 1_{rb}^{(kl)} \cdot \vec{r} + N_{rb}^{(kl)}, \quad (7)$$

where  $1_{rb}^{(kl)} = (1_r^{(k)} - 1_b^{(k)}) - (1_r^{(l)} - 1_b^{(l)})$  is the difference between the rover-satellite vector k and rover-satellite vector l as illustrated in figure 1.

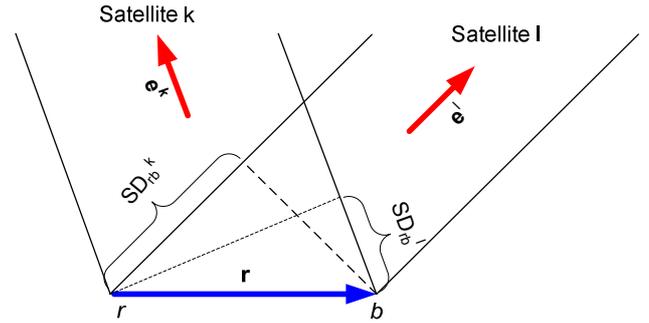


Figure 1: Double difference vector formation

For the (n-1) double difference measurements, we can generalize this expression as a linear model of observations:

$$DD = H \cdot X, \quad (8)$$

with H :

$$H = \begin{bmatrix} h_{float} & 0 & 0 & 0 \\ h_{float} & 0 & 0 & 0 \\ h_{float} & I_{nxn} & 0 & 0 \\ h_{float} & 0 & I_{nxn} & 0 \\ 0 & 0 & 0 & h_{float} \\ 0 & 0 & 0 & h_{float} \end{bmatrix} \quad (9)$$

$h_{float}$  is a nx3 matrix representing the double difference vector  $1_{ur}^{(kl)}$  and  $I_{nxn}$  the nxn identity matrix. This observation matrix enables a linear model of the observations.

It is now possible to process the classic Kalman filter equations:

- Estimation:

$$\begin{aligned} X_{est} &= F \cdot X, \\ P_{est} &= (F \cdot P \cdot F^T) + Q, \end{aligned} \quad (10)$$

where F represent the transition matrix, and P is the covariance estimate of the system. The matrix Q represents the covariance of the functional model.

- Computation of the Kalman gain :

$$\begin{aligned} DD_{est} &= H \cdot X_{est}, \\ K &= P_{est} \cdot H^T (H \cdot P_{est} \cdot H^T + R)^{-1}, \\ Y &= DD_{est} - DD, \end{aligned} \quad (11)$$

where R represents the estimates covariance of the measurements. In the above equation (11), Y denotes the vector of residuals of the observation measurements.

- Update :

$$\begin{aligned} P &= P_{est} - K \cdot H \cdot P_{est} \\ X &= X_{est} + K \cdot Y. \end{aligned} \quad (12)$$

The DDA estimates in the filter are float ambiguities, which means it is only approximation of the real integers values. To round the ambiguities values to these real values and achieve the centimetre level desired, we need a method to estimate the real integer value of the DDA.

The satellite management allows the software to run in real-time. Since no satellite visibility is known a priori, we need to enable an intelligent satellites constellation management. Also an atmospheric correction can improve the general accuracy.

### 3.2 Satellites constellation management

In a real-time test environment, the visibility and availability of the GPS satellites are subject to changes. Some measurements can disappear for a certain number of epochs and some new satellites can be part of the selection.

We choose the satellites constellation towards an intelligent satellite selection (ISS) with classic criteria: a GDOP under 12, an elevation superior of 12 degrees and a lock time (representing the time of tracking) up to 100 epochs. We incorporate in the solution each visible satellite for L1 and L2 frequencies.

During the Kalman filtering, only the matrix P and the vector X are used epoch by epoch. The other matrices are computed toward the number of satellites selected at each epoch. As a consequence, we only need to manage those two quantities in order to adapt the Kalman filter with the satellites constellation change.

If a GPS satellite is removed from the selection, we simply delete the line and row corresponding to this satellite. This does not induce an error in the solution since all the covariance parameters of the others satellites stay in the solution. If a satellite is added in the solution, we therefore add a new line and a corresponding row with initialized covariance parameters. For the DDA of the new satellite, we initialize it with each ambiguity calculated as the difference between the pseudo range and the carrier phase measurement.

When a satellite becomes available again, the corresponding ambiguities have also changed, due to a loss of carrier phase tracking. After a basic initialization based on pseudo range, the ambiguity vector is computed with management of the covariance matrix P.

### 3.3 Atmospheric corrections

The ionospheric correction is made through the ionosphere model, using the two available frequencies L1 and L2 for the Novatel receivers. It uses a linear combination of these two signals. The double differences iono-free pseudo range and the double differences iono-free ADR lie as follow [8]:

$$\rho_{IF} = 2.546\rho_{L1} - 1.546\rho_{L2} \quad (13)$$

$$\begin{aligned} DD\phi_{IF} &= 2.545(DD\phi_{L1} - \lambda_1 \cdot N_{rb_{L1}}^{(kl)}) \\ &- 1.546(DD\phi_{L2} - \lambda_2 \cdot N_{rb_{L2}}^{(kl)}) \end{aligned} \quad (14)$$

The double differences iono-free ADR can be computed only when the DDA of L1 and L2 are resolved or directly in the Kalman filter. For more details on ionospheric corrections, the reader can refer to more specific paper [1].

The implemented tropospheric model used is the Saastamoinnen model, with standard atmospheric parameters in the local region [8].

## 4. AMBIGUITIES RESOLUTION OF THE CARRIER PHASE MEASUREMENTS

The LAMBDA method has been introduced by the University of Delft in 1993 and is widely used to estimate the least square estimation of the DDA.

As detailed in [3], [14], the principle of the LAMBDA method is to find the integer least square estimate with a discrete search that satisfy

$$(\hat{a} - a)Q_a^{-1}(\hat{a} - a) < \chi^2, \quad (15)$$

where  $\chi$  is the bound value found for the space search of the ambiguities, using the covariance distribution that determine the ellipsoid space and the number of required candidates. The matrix  $Q_a$  is extracted from the matrix P in the Kalman filter, and represents the variance of the

ambiguity estimates. In the LAMBDA method, the matrix is decorrelated to improve the search volume of the ambiguity candidates.

To validate the estimates of the LAMBDA method, we use the Fisher-ratio test as in [17]:

$$\rho = \frac{t(\tilde{a}_2)}{t(\tilde{a}_1)}, \quad (16)$$

where  $t(\tilde{a}_i) = \|\hat{a} - \tilde{a}_i\|_{Q_i^{-1}}^2$  is the square norm of the  $i$ -th estimates. Results show that if this ratio between first and second candidates is up to 2, the first estimate is valid and then incorporate in the solution. The LAMBDA method is launched at each epoch. In order to improve the search algorithm and the validity of the estimates, a correct evaluation of the process noise variance and the measurements noise variance has to be taken into account in the Kalman filter. A robust method of these two estimates is presented in the next section.

## 5. ROBUST FILTERING FOR HIGH-PRECISION RTK POSITIONING

Most of existing works on robust estimation for GPS use the M-estimation method to estimate the covariance matrix of the measurements [7], [15]. Some robust estimator can also detect and correct outliers in the measurements [9]. However, this is not the only issue that challenge the RTK systems in long-baseline or severe navigation conditions. Other non-usual errors may include satellites failure, multipath, interference and jamming, software convergence and breakdown problems.

In this section, we develop a new adaptative robust filtering based on the robust M-estimation approach [2]. This method uses a weighting function to adapt and correct the contribution of the updated parameters in the Kalman filter. It serves to measure the accordance of the discrepancy between the updated parameters. Furthermore, it is an efficient way to include proper weights for individual observations at each epoch to mitigate the effect of measurements outliers, model mismatching and impulsive estimation errors.

The least-square method estimation or the Kalman filter as its recursive version minimizes the sum squares of measurements errors:  $\sum_{i=1}^n \|DD_{est} - DD\|^2$ . The M-estimation

method minimizes the sum of a function  $\rho$  of residuals:

$$\sum_{i=1}^n \rho(Y_i) = \sum_{i=1}^n \rho(DD_{est} - DD) \quad (17)$$

The choice of this function  $\rho$  is related to the statistical distribution of the residuals vector  $Y$ . We propose a robust estimator based on an analysis of the histograms of the measures [10]. The robust estimation method is used

to estimate the covariance of the measurements matrix  $R$ . This matrix is computed regarded to classic outliers, which could appear in the residual of the measurements DD. In this case, a robust estimation of the measurements is made and the covariance matrix of the process can be estimated. In summary, we introduce a modified Kalman filter with optimal detection of cycle slips and code outliers, and navigation parameters estimation.

In brief, the computation of the robust covariance estimate is equivalent to multiply the measurements covariance with a weighting matrix  $W$  [9]. The matrix  $W$  is defined as a diagonal matrix  $W = \text{diag}(e_i)$ , where the diagonal elements are defined as follows,

$$W(e_i) = \begin{cases} 1 & \text{if } |e_i| < a \\ a & \text{if } a < |e_i| < d \\ \frac{a}{d} \exp(1 - \frac{e_i^2}{d^2}) & \text{if } |e_i| > d \end{cases} \quad (18)$$

The parameters  $a_i$  and  $d_i$  are evaluated for each matrix with the variance of the residual's measurements. We use the measurement's histogram to determine the correct values of the outliers [10]. As the distribution of the covariance matrix of Gaussian measurements is a chi-square distribution, one may use this statistical test to detect impulsive update in the Kalman filter [15].

We estimate the factor  $a$  as the medium value of the outliers in the histograms method. The outlier represents the value outside the 90% normal range. For this test, we find a medium value for  $a$  of 5 for the pseudo range, and 0.5 for the ADR and the Doppler. According to extensive simulations and tests, we choose  $d = 2a$  in our implementation.

Using this weighting matrix, we can update the Kalman filter for the measurements and process variance respectively:

$$\begin{aligned} Y &= W \cdot Y \\ R &= W^{-1} \cdot R \cdot W^{-1} \end{aligned} \quad (19)$$

Similarly, we can also update the covariance of the process covariance matrix  $Q$ , as

$$Q = W \cdot Q \quad (20)$$

The proposed procedure is different from functional model-error compensation and outliers mitigation methods. It changes the covariance matrix or equivalently the weight matrix of the predicted parameters to cover the model errors, measurements outliers and the estimation errors (e.g. breakdown of the software in a specific step). This scheme can not only resist to the influence of impulsive kinematic model and measurements errors, but also controls the effects of observations noise. The main observation is that the use of robust approaches without

errors control may drives the software to be robust but not optimal. Consequently, if the included statistical test step indicates a normal condition with Gaussian noise, it would be better to use the standard Kalman filter based on the least-square method which is optimal in the sense of maximum-likelihood estimation.

To test the robust estimator, we add to the measurements an impulsive non-Gaussian noise, which could represent outliers or interference, at each 200 epoch in the carrier phase measurement of a satellite. As shown in Figure 2, this noise introduces some spikes in the solution. Furthermore, the ambiguities resolution is inaccurate for some period between the outliers. This kind of noise largely biases the LAMBDA resolution method.

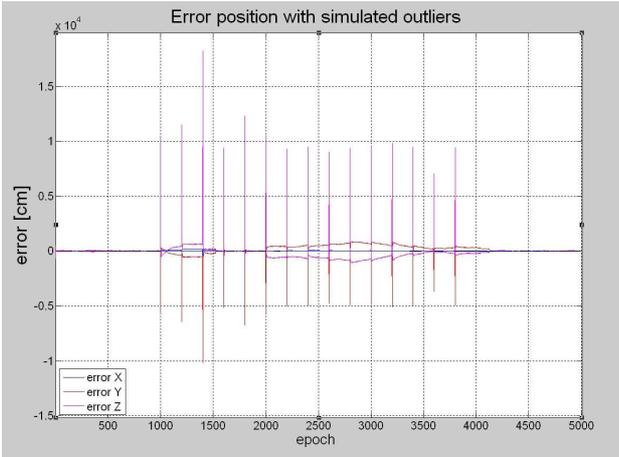


Figure 2: error positioning with classic Kalman filter

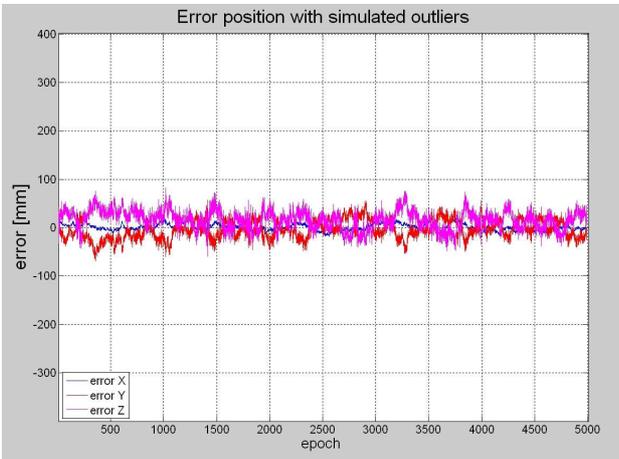


Figure 3: error positioning with robust Kalman filter

	Classic Kalman filter			Robust Kalman filter		
	x	y	z	x	y	z
Medium (cm)	10.6	67.5	-94.4	0.12	1.58	1.77
Variance	2.9e4	3.5e4	9.2e6	0.47	4.96	7.23
% fixed	46.6 %			99.98 %		

Table I: Analysis comparison between classic Kalman filter and robust Kalman filter.

In Figure 3, the results are obtained using the robust Kalman filter version according to equations (19)-(20). This filter uses the robust covariance of the measurements to detect the outliers and correct the solution computation. As a result, the Kalman filter is able to give an accurate position at the centimetre level and to resolve the DDA correctly.

In Figure 4, we face a situation where the increase baseline (40 km) and the multipath biases induce a wrong estimation of the DDA. The solution has a bias due to an ambiguity cycle difference toward the correct fixed DDA. The statistical test step detected easily the presence of divergence errors. Then when we use the robust method and the weight matrix covers the errors and mitigates the estimate state covariance. The LAMBDA method continues to provide the right DDA. In the figure 4, we see that there is no jump in the solution since the DDA keeps its stability.

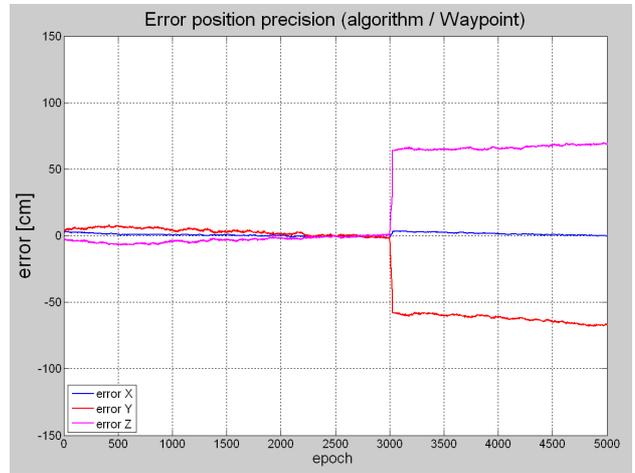


Figure 4: error positioning with classic Kalman filter

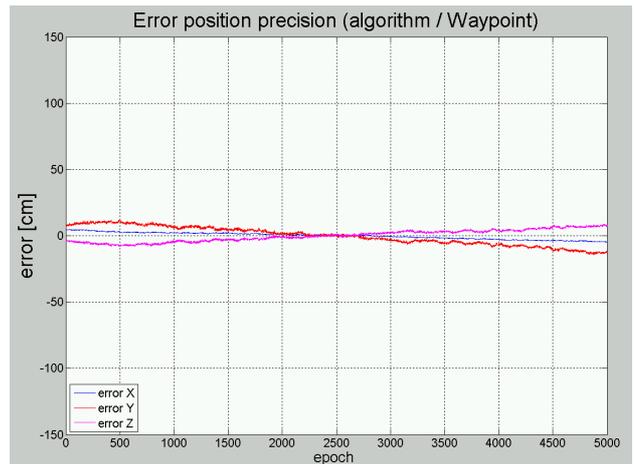


Figure 5: error positioning with robust Kalman filter

This results show that in some severe navigation conditions, a robust estimation of the covariance of the measurements is necessary to keep a good ambiguities resolution. Indeed, some improvements can be made to have a robust method of ambiguity resolution for long baseline.

Including the method of robust Kalman filtering and state parameters estimation, a complete software has been developed with all the different functionalities presented. The next sections will show how the user can handle the complete software with a GUI and how we can obtain the results for different tests.

## 6. GRAPHIC MONITORING OF THE SOFTWARE

The laboratory developed a graphic interface associated with the software. This interface allows the user to have a complete visibility of every real data process by the algorithm and to be able to select the suitable criterion for a good software management.

One can choose the GPS frequency used in the solution, the type of double difference made in the Kalman filter, widelane, narrowlane or ionosphere free. We can also choose to resolve the carrier phase ambiguities or not.

The software gives the possibility to import text data, like waypoint solution, and process the difference between the two positions. All this selection and the different selection are useful to estimate the impact of the different parameters used in the solution. For example, the atmospheric correction or the lock time can be activated quickly in the interface as illustrated in Figure 6.

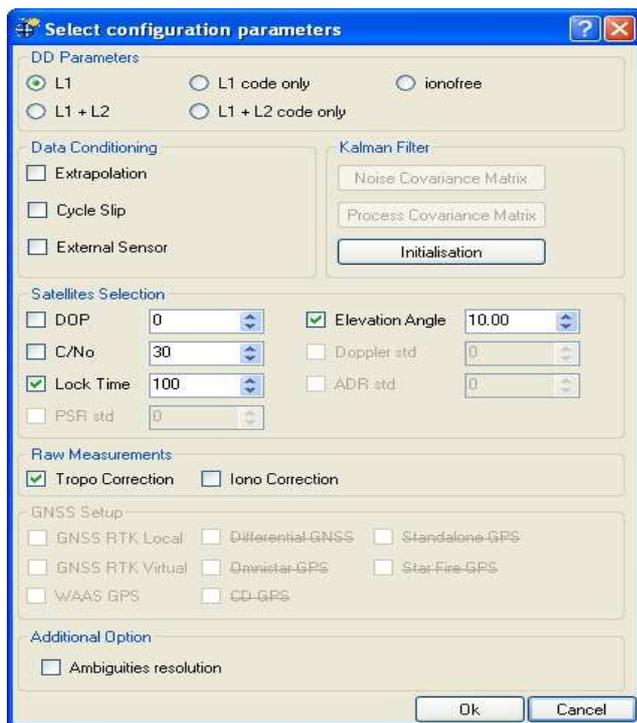


Figure 6: Example of the RTK criteria selections

The flexibility of the interface and the code implementation allows the user to incorporate more and more flexibility for new criteria as the software develops new features.

All the data coming from the RTK algorithm are available to the user. Solution computation like the user position can be displayed in different windows. The satellites constellation is also shown. All the

measurement can be seen, the number of satellites used and the corresponding DOP. The interface can also display the different matrix epoch by epoch computed in the Kalman filter. All the different values can be displayed in a graphic window, like the ambiguities computed by the software as shown in Figure 7.

In order to demonstrate the robustness of the developed algorithm, different tests in real time were done with a van in a parking lot on the 5th October 2007. The chosen environment allows a clear open sky and the multipath was negligible. The data have been recorded with 2 Novatel receivers, one as the rover in a van and the other as the base station (arrow on Figure 8). The data can be post processed in the same way and accurate position is seen as in a real-time scenario.

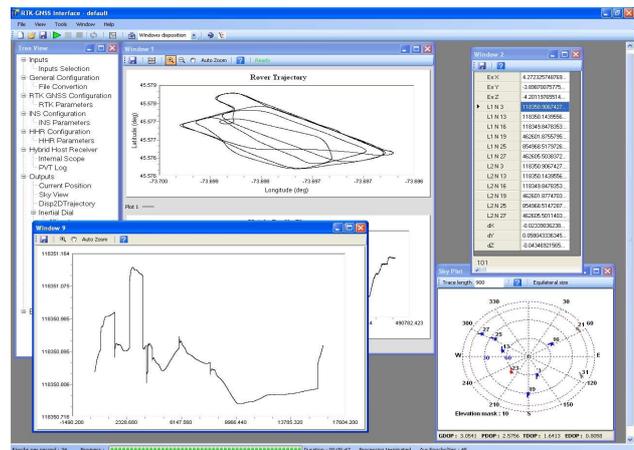


Figure 7: Graphic interface of the RTK software

## 7. PERFORMANCE ANALYSIS OF THE RTK SOFTWARE WITH NOVATEL RECEIVERS

In this test, real data recorded by two different receivers were post-processed by the robust RTK algorithm. In real-time, we process epoch by epoch the Kalman filter, which gives the accurate position. The ISM allows the software to be able to manage different situation of satellite visibility and availability and still give an accurate position in real-time.

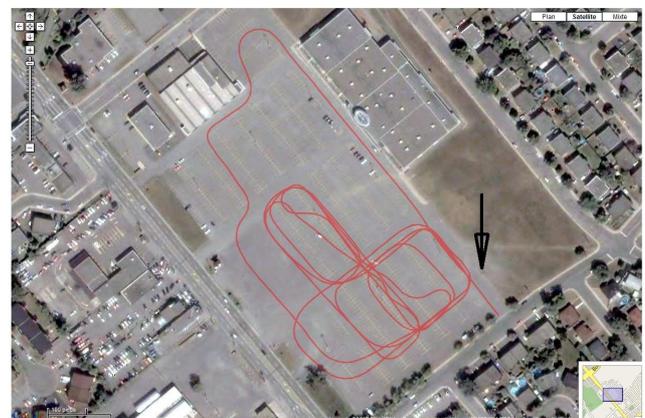


Figure 8: Trajectory of the rover and the base (arrow).

The next figure shows the position precision obtained with the developed RTK software. This solution is

compared with the post-processing solution of the software GRafnav of Waypoint Solutions, which gives precision below 10 cm [16]. We used the robust covariance estimation of the residuals to detect any possible outliers or jumps in the ambiguities resolution via LAMBDA method.

The Figure 9 shows the errors on the three axis x, y and z. The data corresponds to approximately 25 min of recording. The standard deviation is 1.1 cm on x-axis, 2.5 cm on the y-axis and 2.6cm on the z-axis.

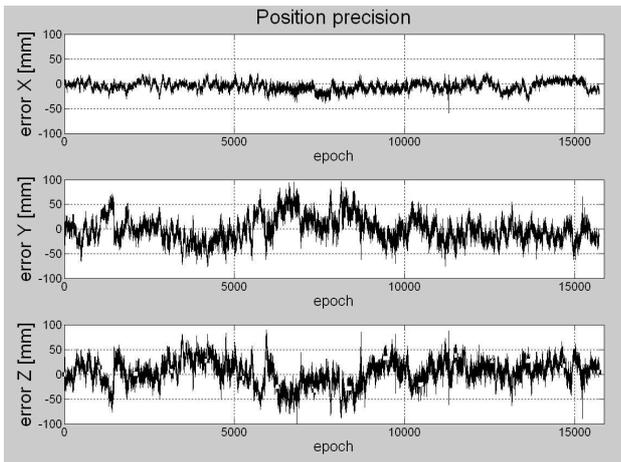


Figure 9: Error positioning with the Novatel receivers

As shown in the Figure 10, the constellation of satellites varies from 5 to 7 in the solution. The changes of satellites can induce some peaks in the solution before the convergence of the filter and the ambiguities resolution.

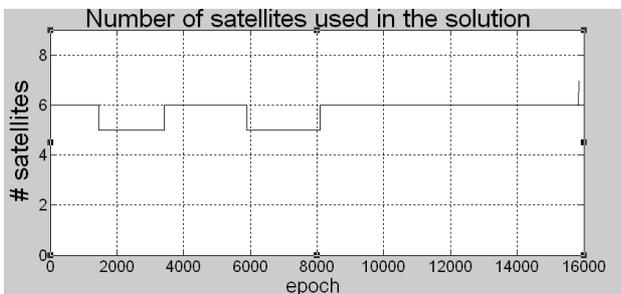


Figure 10: Number of satellites used in the solution

The precision shows that the ambiguities are resolved correctly, at least like the waypoint software. This is due to the correct integrity of the data and the accuracy of the LAMBDA method solution, once the Kalman filter solution is robust. Indeed, the Kalman filter algorithm feeds the float solution to the LAMBDA method, thus more precise this initial solution is, and more the LAMBDA method will have a precise and small space search of ambiguities. This is an essential pre-processing to increase the efficiency of LAMBDA. This short baseline test showed that the residual's measures needs to be correctly evaluated and the management of the covariance matrix improves the general performance of RTK.

## 8. RTK PERFORMANCE USING LACIME RECEIVER

The LACIME laboratory has developed a complete software defined radio (SDR) GNSS receiver based on FPGA [13]. The platform currently processes the L1 band (i.e. 1575.42 MHz) GPS and Galileo with WAAS capability. The FPGA is used to perform the signal acquisition, tracking, demodulation as well as the message decoding. The code and carrier measurements and navigation messages are further sent to the PC through the Compact PCI bus signals for post-processing.

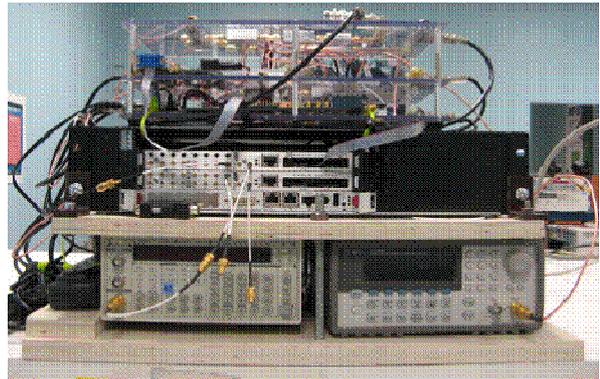


Figure 11: The GNSS SDR receiver

The main advantage of this platform for research study is its flexibility provided by the software. This low cost and rapid development approach is appropriate for taking into account all the new GNSS signals.

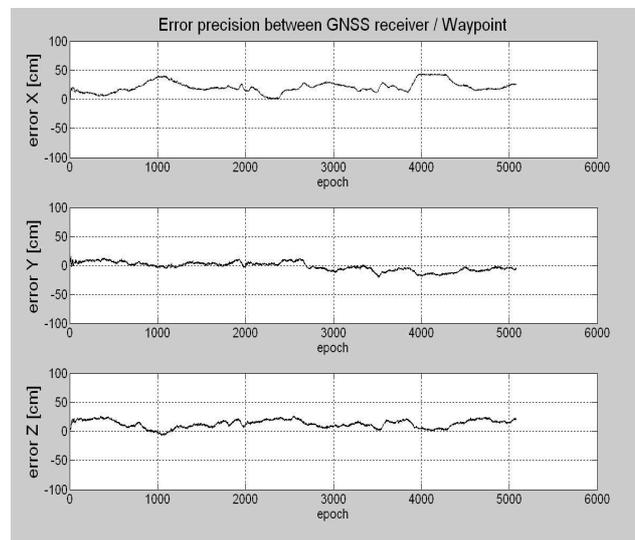


Figure 12: Error positioning for GNSS receiver

Figure 12 presents the results in terms of precision for the 20 min test, compare with the same Waypoint post process results as performed for Figure 9. The results are encouraging for further tests and software implementation. The precision is lower than for the Novatel receiver, but a better signal processing has to be made on the measures before computing, to improve the solution. The ambiguity problem has been resolved in the same way with the Kalman filter and the LAMBDA

method, giving a position precision of approximately 10 cm RMS.

## 9. CONCLUSION

The purpose of this study is to be able to provide real-time position of the airborne in a robust and reliable way, with the accuracy of classic RTK algorithm. The centimeter level of accuracy can be achieved for medium baseline and will be very helpful for further developments in long range RTK, since the precision of the gravimeter needs such accuracy in position but also acceleration.

The developed Kalman filter uses dual frequency pseudo-range measures, carrier phase measures and Doppler measures and the filter provide the float DDA estimate in real time. This estimate as its correlated variance is used in the LAMBDA method to fix the ambiguities and achieve the centimeter precision. A robust measurement covariance matrix is performed to insure the robustness and the validity of the fixed ambiguities, despite the presence of severe noise or outliers or estimation errors.

We developed a fast algorithm in C code, which allows the user to compute and analyze large data files, and to work in real time. A graphic interface monitor has been developed to allow the user to control every criteria of the algorithm and to have access to all the results computed by the software in a graphic and friendly interface.

The results show an accurate algorithm, with centimeter precision. Also, the studies of the precision of the solution show that the LAMBDA method is performed in a robust and reliable way without sudden jump in the solution.

The development of the algorithm will be follows by the graphic interface development. More parameters and ambiguities resolution will be available, also with a more accurate visibility of the robustness of the algorithm. Also accurate model of errors will be developed to extend this algorithm for long baseline applications.

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