Multipath Mitigation Techniques Using Maximum-Likelihood Principle

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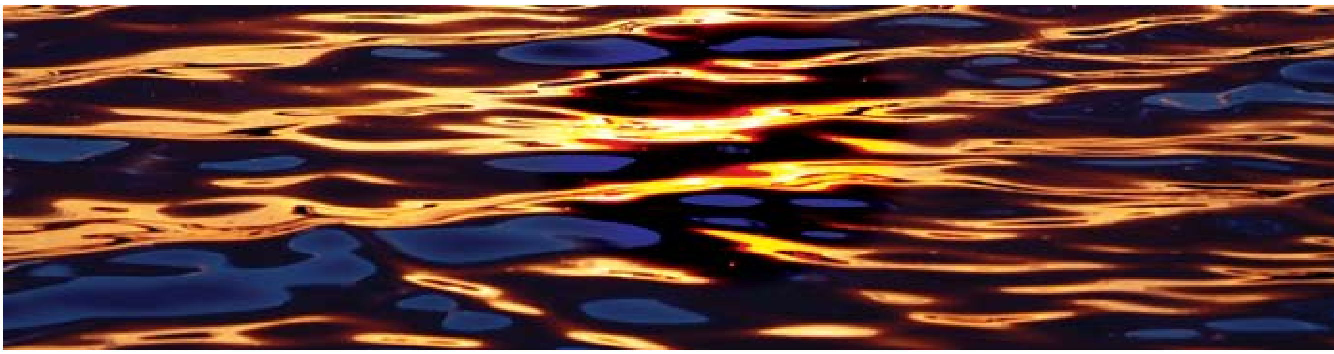

With increased computer power, receiver designers can now make use of complex algorithms and computationally intense solutions to reduce the bad effects of multipath – reflected signals – on GNSS equipment. The authors describe a category of multipath mitigation techniques based on principles of maximum-likelihood (ML) estimation — including a new variation of their own — that can improve receiver performance.

Although modern GPS receivers achieve high pseudorange accuracy in line-of-sight (LOS) conditions, multipath remains a dominant source of ranging error in GNSS.

Multipath interference occurs when the user device receives reflected signals in addition to the direct LOS signal. These interference signals are generally reflected from the ground, buildings or trees in terrestrial navigation, while signal reflections from the host-vehicle body are more common in airborne and marine applications.

Two kinds of multipath exist: *specular* multipath arising from discrete, coherent reflections from smooth surfaces such as standing water, and *diffuse* multipath arising from diffuse scatterers and sources of diffraction. (The visible glint of sunlight off a choppy sea is an example of diffuse multipath.)

Multipath signals are generally considered undesirable in the GNSS realm because they destroy the correlation function shape used for time delay estimation, but can be useful in some cases (for example, for acquisition). Although some wireless communications techniques exploit multipath to provide signal diversity, the key point in GNSS is to efficiently mitigate the multipath effect because we use only the satellite-receiver transit time offset of the LOS signal for positioning.

This article will discuss a category of multipath mitigation techniques a principle known as maximum-likelihood estimation, reviewing some of the leading examples introduced over the last 15 years or so and then describing a new ML approach based on what we call the Fast Iterative Maximum-Likelihood Algorithm (FIMLA).

Multipath Mitigation

Examples of recently developed in-receiver multipath mitigation methods include the narrow correlator, the strobe correlator, the Multipath Estimating Delay Lock Loop (MEDLL), Multipath Elimination Technology (MET), the Multipath Mitigation Technology (MMT), and the Vision Correlator (VC).

Spatial processing is another class of multipath mitigating technology that includes choke-ring antenna design and directive antenna arrays. Directive antennas are generally physically large and heavy and are not affordable for most of civilian applications. This class of multipath mitigation will not be discussed further in this article.

The actual multipath performance of a given signal and receiver combination depends on various parameters of both, including the signal-type modulation, code chipping rate, the pre-correlation bandwidth and filter characteristics, the number of received multipath signals, the relative power of multipath signals, the path-delay, chip spacing between correlators, and the type of discriminator and algorithm used for code and carrier tracking.

Maximum-Likelihood Methods

Several multipath mitigation methods introduced in recent years such as MEDLL, MMT and VC rely on maximum-likelihood (ML) estimation principles and are driven to approach theoretical performance limits. This is not surprising, because ML estimators have desirable statistical asymptotic properties.

The idea behind ML estimation in general is to determine the parameters that maximize a likelihood function, which is the joint probability density function (PDF) of the sample data. This estimation method does not require *a priori* information and assumes that the unknown parameters are constant over an observation period, typically hundreds of milliseconds or multiple seconds for high-sensitivity receivers. Thus, ML offers the optimal approach in many practical situations when the prior knowledge needed for Bayesian estimators, such as maximum a posteriori (MAP) and minimum mean-square error (MMSE) estimation, is not available.

ML Simple, But Complex

Although the methodology for ML estimation is simple, the implementation is mathematically intense.

Using the interesting progress in the branch of optimization and today's computer power, however, complexity is no longer a significant obstacle. ML-type tracking loops are typically complex and difficult to implement, as they require the receiver to measure the received signal cross-correlation function for each reflected path with multiple correlators and to process these measurements with complex algorithms.

One commonly used technique, a line search approach, is applied to find the estimates that maximize the log-likelihood. Recently, Lawrence Weill, inventor of MMT, applied a nonlinear transformation on the multipath parameter space to reduce the computation load of the likelihood function maximization. M. Z. Bhuiyan et al. has published a non-coherent version of MEDLL that generates phases as a random uniformly distributed parameter and chooses the one that minimizes the mean square error of a residual correlation function. (See Additional Resources section at the end of this article.)

The latest ML multipath mitigation approach is the Fast Iterative Maximum-Likelihood Algorithm (FIMLA) developed by M. Sahmoudi that uses a GNSS signal model structure and the spreading code periodicity. With these, FIMLA develops an efficient implementation of the Newton iterative likelihood-maximization method by finding simple analytical expressions for the first and second derivatives of the likelihood function. Later in this article we review and discuss these different implementations of ML multipath estimation in terms of how they each improve processing efficiency.

Multipath Signal Model

Considering a received multipath or M – path signal composed of the LOS and (M – 1) reflected paths, we can express the received complex baseband signal at the input of a GPS receiver as,

$$r(t) = \sum_{i\ 1}^{M} A_i c(t-\tau_\iota)e^{j\phi_\iota} + n(t) \quad (1)$$

where (A_i, τ_t, ϕ_t) are the amplitude, delay, and phase of the i-th signal path, which are assumed to be constant over the period of observation. The code modulation signal is denoted by *c(t)*, and *n(t)* is the zero-mean complex Gaussian noise.

It is assumed that the signal in equation is stripped of navigation data modulation by existing data demodulation methods. In the presence of one multipath (i.e., M=2), we can express the Doppler-compensated baseband signal by the following insightful and simple model, to which we will refer in later discussions

$$r(t) = A_1 c(t-\tau_1)e^{j\phi_1} + A_2 c(t-\tau_2)e^{j\phi_2} + n(t) \quad (2)$$

Multipath Estimation Using ML

The idea of parametric multipath mitigation methods is to reduce the multipath effect by estimating both the LOS $A_1 c(t-\tau_1)e^{j\phi_1}$ and multipath contribution:

$$\sum_{i=2}^{M} A_i c(t-\tau_\iota)e^{j\phi_\iota}$$

During signal parameter estimation, the reconstructed multipath component is subtracted from the received signal to obtain a good estimate of the LOS, such that a very precise estimate of the LOS time-delay, , τ_1 can be computed.

Thus, the multipath mitigation problem is formulated as a statistical estimation problem of the unknown parameters θ = (A_1, A_2, .. , A_M, τ_1, τ_2, .. , τ_M, ϕ_1, ϕ_2, .. , ϕ_M). According to ML estimation theory, the best estimates are those values that maximize the likelihood function, that is, the joint data PDF,

$$P(r;\theta) = c \exp\left\{-\frac{1}{2\sigma^2}\int_{T_0} |r(t) - \hat{A}_1 c(t-\hat{\tau}_1)e^{j\hat{\phi}_1} - \sum_{i=2}^{M}\hat{A}_i c(t-\hat{\tau}_\iota)e^{j\hat{\phi}_\iota}|^2 dt\right\} \quad (3)$$

In practice, where the signal is sampled, the integral in (3) can be replaced by a summation over all samples in the considered time interval T_0. This is equivalent to minimizing the mean squared error between the received signal and the estimated version via the log-likelihood cost function:

$$\Gamma(\theta) = \int_{T_0} |r(t) - A_1 c(t-\tau_1)e^{j\phi 1} - \sum_{i=2}^{M}\hat{A}_i c(t-\hat{\tau}_\iota)e^{j\hat{\phi}_\iota}|^2 dt \quad (4)$$

To compute the ML estimates, the partial derivatives of Γ(θ) with respect to each parameter are set to zero and then solved for. This is a nonlinear optimization problem on a 3M-dimensional space spanned by θ, which is computationally extensive. According to (2), we have six parameters, $\theta = (A_1, A_2, \tau_1, \tau_2, \phi_1, \phi_2)$, to be estimated for the case of one multipath. To deal with this problem, many algorithms of the ML multipath estimation have been introduced since 1992.

MEDLL

To illustrate how ML-based estimation works in a bit more detail, we first consider the Multipath Estimating Delay Lock Loop, which is an efficient statistical approach for multipath mitigation . Implemented commercially in 1995, this algorithm became the first widely known and practical method for multipath mitigation.

MEDLL is still in use today. It improves the C/A-code narrow-correlator performance by confining the residual pseudorange error to a smaller region of secondary-path relative delay out to approximately 30 meters (i.e., the second path is less than 30 meters longer than the direct path). Within this range, the residual error is reduced to approximately 5 meters worst-case with a one-half amplitude secondary-path signal when the receiver bandwidth is 8 MHz.

To understand this technique, we will briefly summarize the theory behind MEDLL. Setting the partial derivatives of (4) with respect to the signal parameters to zero yields a set of nonlinear equations. To overcome the difficulty of solving these equations, MEDLL approximates the overall cross correlation function using a set of reference correlation functions with certain delay, phase and amplitude,

$$R_{rc}(\tau) = \sum_{i=0}^{M} R_i(\tau) \quad (5)$$

where $R_i(\tau)$ is the component of $R_{rc}(\tau)$ corresponding to the i-th path.

In the MEDLL approach, $R_{rc}(\tau)$ is computed at delays $\tau = k\Delta\tau$ in a parallel bank of correlators. The cross-correlation values $R_{rc}(k\Delta\tau)$ are the input to the digital signal processor (DSP), which solves the MEDLL equations.

Indeed, the inventors of this approach originally proposed an interference cancellation technique that reduces the complexity, which can be summarized as follows:

- *Step 1- Initialization:* Calculate the correlation function $R_{rc}(\tau)$, find the maximum (called peak 1), and its corresponding delay $\hat{\tau}_1$, amplitude $\hat{A}_i$ and phase $\hat{\phi}_1$.
- *Step 2- Successive multipath correlations cancellation:* Subtract the contribution of the calculated peak to yield a new approximation of the correlation function,

$$R^{(1)}_{rc}(\tau) = R_{rc}(\tau) - \hat{A}_1 R_c(\tau - \hat{\tau}_1)e^{j\hat{\phi}_1} \quad (6)$$

and find the new peak (peak 2) of the residual correlation function $R^{(1)}_{rc}(\hat{o})$ and its corresponding delay $\hat{\tau}_2$, amplitude $\hat{A}_2$ and phase $\hat{\phi}_2$. Subtract the contribution of peak 2 from $R_{rc}(\tau)$ and find a new estimate of peak 1.

- *Step 3- Convergence:* Repeat step 2, until a certain criterion of convergence is met.

The experimental results presented by the inventors of

MEDLL show that MEDLL reduces the effects of one reflected multipath signal by up to 90 percent over a standard narrow-correlator receiver, as illustrated in **Figure 1** which assumes an 8 MHz bandwidth and single reflection with half the amplitude of the direct signal. As shown, MEDLL eliminates any multipath biases for delays larger than 0.1 chip and has better performance than standard wide and narrow correlators.

Although MEDLL requires a large number of correlators and large algorithmic computations, it was an important evolutionary step in the receiver-based battle against multipath. In addition, MEDLL has stimulated the design of many algorithms for the implementation of the ML-based multipath mitigation, which we will discuss next.

A Non-Coherent MEDLL Algorithm for ML Multipath Estimation. Recently, M. Z. Bhuiyan and others at Finland's Tampere University of Technology suggested a non-coherent implementation of MEDLL to reduce the parameters space of optimization. This is done by including additional non-coherent integrations in the likelihood cost function to be optimized.

As the phase information is lost by the squaring operator (in the non-coherent integration process), they compensate for it by generating random uniformly distributed phases over $[0,2\pi]$ and choosing the value corresponding to the minimum mean square error of the residual correlation function, i.e., the difference between the computed correlation and the reconstructed one using the reference code correlation and the estimated amplitude, delay, and phase. Thus, the parameter space has been reduced since it does not include the phases.

The performance of this approach depends on the number of random phases considered; so, the larger this number is, the better the performance is — although at the cost of increased processing requirements.

MMT Algorithm for ML Multipath Estimation. Developed for the case of $M = 2$ by Weill and Ben Fisher of Comm Sciences Corporation (CSC), MMT uses a nonlinear transformation on the multipath parameters space to permit rapid computation of a 2-path log-likelihood function that has been partially maximized with respect to four new parameters — reflected in the transformation (8) —instead of the six multipath parameters. The final maximization requires a search in only two dimensions, aided by acceleration techniques.

To further increase computational efficiency, MMT operates on a data vector of small dimensionality, obtained by a proprietary method for lossless compression of the raw signal observation data. As presented in Weill's original paper cited in Additional Resources, we consider only one reflected path superimposed on the LOS signal as given by equation (2).

This complex baseband signal can be separated into its real and imaginary components; *x(t)* and *y(t)*, respectively. The log-likelihood function of the six parameters to be estimated is

$$\Gamma = \int_{T_0} [x(t) - A_1 c(t-\tau_1)\cos(\phi_1) - A_2 c(t-\tau_2)\cos(\phi_2)]^2\, dt + \int_{T_0} [y(t) - A_1 c(t-\tau_1)\sin(\phi_1) - A_2 c(t-\tau_2)\sin(\phi_2)]^2\, dt. \quad (7)$$

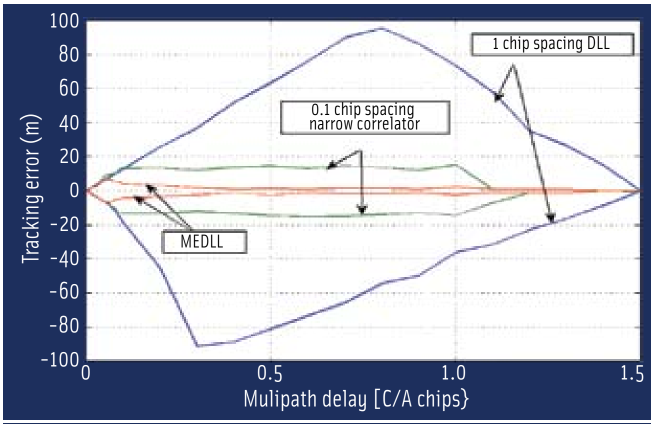

FIGURE 1 LOS code tracking error versus the relative multipath delay $(\tau_2 - \tau_1)$

To minimize the preceding log-likelihood function, a major simplification can be achieved by applying the nonlinear and invertible transformation,

$$a = A_1\cos(\phi_1),\ b = A_2\cos(\phi_2),\ c = A_1\sin(\phi_1),\ d = A_2\sin(\phi_2) \quad (8)$$

When this transformation is applied and the integrals in are expanded, the problem becomes one of minimizing,

$$\begin{aligned}\Gamma = &\int_{T_0} [x^2(t) + y^2(t)]dt + (a^2 + b^2 + c^2)R_c(0)\\ &- 2aR_{xc}(\tau_1) - 2bR_{xc}(\tau_2) + 2abR_c(\tau_1 - \tau_2)\\ &- 2cR_{yc}(\tau_1) - 2dR_{yc}(\tau_2) + 2cdR_c(\tau_1 - \tau_2).\end{aligned} \quad (9)$$

This cost function is now quadratic in *a, b, c* and *d*; so, by setting the partial derivatives with respect to these parameters to zero, the result is a linear system. For each pair of values τ_1 and τ_2, this linear system can be explicitly solved for the minimizing values of *a, b, c* and *d*. Then the search space is reduced from six dimensions to two dimensions.

The MMT algorithm can be summarized as follows,

- *Step 1- Search in the* (τ_1, τ_2) *domain:* at each point find the values of *a, b, c* and *d* which minimize Γ at that point.
- *Step 2-* Identify the point $(\tau_1, \tau_2)_{ML}$, where the smallest of all such minima is obtained, as well as the associated minimizing values of *a, b, c* and *d*.
- *Step 3-* Compute the estimates $\hat{A}_{1,ML}$, $\hat{A}_{2,ML}$, $\hat{\phi}_{1,ML}$ and $\hat{\phi}_{2,ML}$ according to Eq. (1.7) by using the inverse of the proposed transformation (8).

MMT was used as the fundamental multipath mitigation approach in the Vision Correlator. Further hardware improvements and optimizations were developed at a manufacturer in the course of commercializing this technology, and the company has licensed all commercial rights to the related patents and works.

FIMLA

The FIMLA algorithm reformulates the cost function as a complex amplitude $a_i = A_i e^{j\hat{\phi}_1}$ for more compact and general derivations. Thus, the phase information is considered implicitly in order to reduce the computational burden.

FIMLA also exploits the periodicity of the GNSS signal code to simplify the log-likelihood function (equation 4) as,

$$\Gamma = -\int_{T_0} |r(t)|^2 dt + 2\Re\left\{a_1^* R_{rc}(\tau_1)\right\} - T_0|a_1{}^2| + 2\Re\left\{a_2^* R_{rc}(\tau_2)\right\} - T_0|a_2|^2 - 2\,\Phi(\tau_2 - \tau_1)\,\Re\left\{a_1 a_2^*\right\}, \quad (10)$$

where $\Re\{.\}$ denotes the real part of a complex number and $\Phi(.)$ is the ideal GNSS-code autocorrelation function over the observation period. To compute the ML estimates, the partial derivatives of Γ with respect to each of the four parameters are set to zero.

Unlike the amplitude parameters, which have an explicit solution, there is no explicit solution to the time delay parameters. This is because the cross-correlation function $R_{rc}(\tau)$ depends on the time delay parameter through $c(t - \tau)$, which does not provide any direct expression of the delay. To overcome this difficulty, the ML estimator is implemented iteratively using the Newton method. The final algorithm is given by,

1- Multipath-free model:

$$\hat{\tau}_{k+1} = \hat{\tau}_k - \frac{\Re\left\{R^*_{rc}(\hat{\tau}_k)\dfrac{\partial R_{rc}(\hat{\tau}_k)}{\partial \tau}\right\}}{\Re\left\{R^*_{rc}(\hat{\tau}_k)\dfrac{\partial^2 R_{rc}(\hat{\tau}_k)}{\partial \tau^2}\right\}} \quad (11)$$

2- Multipath case with one reflected path:

$$\hat{\tau}_{1,k+1} = \hat{\tau}_{1,k} - \frac{\Re\left\{R^{(1)*}_{rc}(\hat{\tau}_{1,k})\dfrac{\partial R^{(1)}_{rc}(\hat{\tau}_{1,k})}{\partial \tau_1}\right\}}{\Re\left\{R^{(1)*}_{rc}(\hat{\tau}_{1,k})\dfrac{\partial^2 R^{(1)}_{rc}(\hat{\tau}_{1,k})}{\partial \tau_1^2}\right\}},$$

$$\hat{\tau}_{2,k+1} = \hat{\tau}_{2,k} - \frac{\Re\left\{R^{(2)*}_{rc}(\hat{\tau}_{2,k})\dfrac{\partial R^{(2)}_{rc}(\hat{\tau}_{2,k})}{\partial \tau_2}\right\}}{\Re\left\{R^{(2)*}_{rc}(\hat{\tau}_{2,k})\dfrac{\partial^2 R^{(2)}_{rc}(\hat{\tau}_{2,k})}{\partial \tau_2^2}\right\}}, \quad (12)$$

where $R^{(1)}_{rc}(\hat{\tau}_1) = \int_{T_0} [r(t) - a_2 c(t - \hat{\tau}_2)]c(t - \hat{\tau}_1)dt$

and $R^{(2)}_{rc}(\hat{\tau}_2) = \int_{T_0} [r(t) - a_1 c(t - \hat{\tau}_1)]c(t - \hat{\tau}_2)dt.$

Naturally, these two correlation functions $R^{(1)}_{rc}(\hat{\tau}_1)$ and $R^{(2)}_{rc}(\hat{\tau}_2)$ represent the correlation with the local code of the received signal after subtracting the estimated second and first paths, respectively. Using FIMLA to estimate the useful LOS parameter τ_1 is both attractive and insightful, as it involves the principle of subtracting the contribution of the undesired multipath signals from the correlation function of the direct path.

This algorithm is easily generalized to the multiple multipath cases by updating each path-delay by the same FIMLA algorithm using the corresponding cross-correlation function after subtracting the other paths' contributions in a sequential procedure. The algorithm aims at keeping the complexity similar to that of the narrow correlator DLL receiver.

Implementation of ML multipath mitigation within existing receiver hardware/components is an important practical consideration because GNSS engineers and companies want to exploit the existing receiver board when they decide to implement a new technique. Indeed, designing and developing new receiver architecture always generates many challenging issues, in addition to the cost constraint.

Surprisingly, the FIMLA can be related mathematically to most of the existing and well-known DLL discriminators, requiring only some additional integrators to compute the additional correlations for the multipath model case. So, the FIMLA algorithm could be implemented in a GNSS receiver as a firmware upgrade or in a software receiver. The correlation between the received data and the code replica is computed at three delay values called early, punctual, and late via the in-phase and quadrature components as

$$R_{rc}(\hat{\tau}_k) = Q_p + jI_p,\; R_{rc}(\hat{\tau}_k - \delta) = Q_e + jI_e,\; R_{rc}(\hat{\tau}_k + \delta) = Q_l + jI_l \quad (13)$$

Using the derivative approximation by finite differences, we can write

$$\frac{\partial R_{rc}(\hat{\tau}_k)}{\partial \tau} = -\frac{1}{2\delta}[(Q_e - Q_l) + j(I_e - I_l)],$$

$$\frac{\partial^2 R_{rc}(\hat{\tau}_k)}{\partial \tau^2} = \frac{1}{\delta^2}[(Q_e + Q_l - 2Q_p) + j(I_e + I_l - 2I_p)] \quad (14)$$

Applying the following approximation, $R_e + R_l - 2R_p = -2\delta\, R_p / T_0$, for both I and Q correlation components,— once on the numerator and once on the denominator of equation with some elementary math equation can be expressed in the following forms

$$\hat{\tau}_{k+1} = \hat{\tau}_k - \frac{T_0}{4}\frac{(Q_e - Q_l)Q_p + (I_e - I_l)I_p}{4(Q_p{}^2 + I_p{}^2)},$$

$$\hat{\tau}_{k+1} = \hat{\tau}_k - \frac{T_0}{2(1 - \dfrac{\delta}{T_0})}\frac{(Q_e^2 + I_e{}^2) - (Q_l^2 + I_l^2)}{4(Q_p{}^2 + I_p{}^2)} \quad (15)$$

These equations are the dot-product power and early-minus late power discriminators, respectively. Thus, FIMLA can use existing DLLs for code tracking. In the multipath case, these discriminators will be applied for each code path delay being tracked, that is, FIMLA applies the DLLs many times to track the multipath signals according to equation , and subtract their effect sequentially as illustrated in **Figure 2**.

In conclusion, a firm comparison between the various ML algorithms is difficult as they have been developed from different perspectives. We can say in retrospect, however, that MEDLL was a pioneering algorithm in multipath mitigation technology, MMT is an economic computational technique, VC is a hardware improvement of MMT, and finally FIMLA is an approach for implementing ML multipath mitigation by extending existing delay-tracking loops.

Other Considerations

The effect of non-white interference on performance of ML-

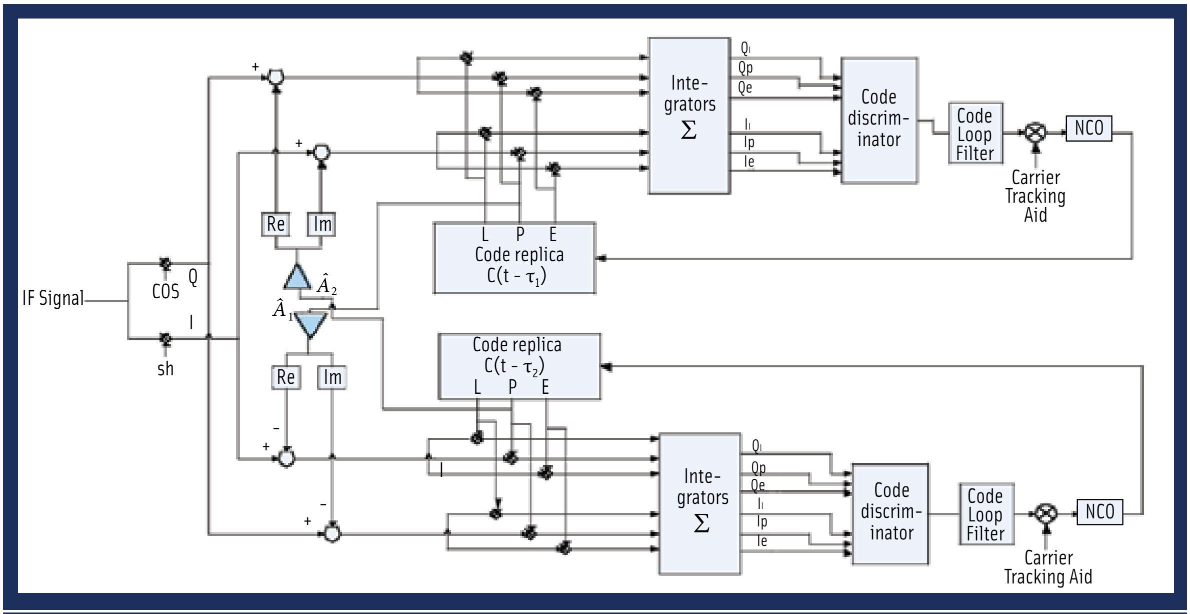

FIGURE 2 FIMLA architecture using multiple of existing DLLs.

based multipath mitigation techniques has not been investigated in the literature but is beginning to receive some attention. Refer to the Additional Resources section at the end of this article for more information.

Finally, the foregoing discussion reviewed approaches that are based on the premise that, over a sufficiently short observation interval, the signal parameters may be viewed as constant, though unknown, quantities. With this assumption, the ML yields the best performance.

However, this assumption is not valid for a large number of observation times. Thus, some kind of Kalman filter tracking algorithms may be employed in conjunction with ML estimation in order to keep the estimates from eventually drifting outside their allowable range.

Manufacturer

MEDLL, MMT, and the Vision Correlator were commercially implemented by **Novatel, Inc.**, Calgary, Alberta, Canada.

Additional Resources

[1] Bhuiyan, M. Z., and E. S. Lohan and M. Renfors, "Code tracking algorithms for mitigating multipath effects in fading channels for satellite-based positioning," *EURASIP Journal on Advances in Signal Processing,* January 2008.

[2] Garin, L., and F. Van Diggelen, and J. Rousseau, "Strobe & Edge Correlator Multipath Mitigation for Code," in *Proceedings of the ION GPS 1996,* Kansas City, Missouri, September 1996

[3] Jones, J., and P. C. Fenton, "The Theory and Performance of NovAtel Inc.'s Vision Correlator", available from <http://www.novatel.ca> or in *Proceedings of the ION GNSS 2005,* Long Beach, California, USA, September 2005

[4] Sahmoudi, M., and M. G. Amin, "Fast Iterative Maximum-Likelihood Algorithm (FIMLA) for Multipath Mitigation in Next Generation of GNSS Receivers," in *IEEE Trans. On Wireless Communication,* Vol. 7, No. 11, November 2008.

[5] Townsend, B. R., and P. Fenton, "A Practical Approach to the Reduction of Pseudorange Multipath Errors in a L1 GPS Receiver", in *Proceedings of ION GPS-94,* Salt Lake City, Utah, USA, 1994

[6] Van Dierendonck, A. J., P. Fenton and T. Ford, "Theory and Performance of Narrow Correlator Spacing in a GPS Receiver", *Journal of the Institute of Navigation,* vol. 39, no. 3, Fall 1992

[7] Van Nee, R. D. J., *Multipath and Multi-Transmitter Interference in Spread-Spectrum Communication and Navigation Systems,* Ph.D. Thesis, Delft University Press, The Netherlands, 1995

[8] Weill, L. R., "Multipath Mitigation Using Modernized GPS Signals: How Good Can it Get?" in *Proceedings of ION GPS 2002,* Portland, Oregon, USA, 2002

For more information regarding the effect of non-white interference on ML-based tracking refer to

Sahmoudi, M., and M. G. Amin (2008), "Robust Tracking of Weak GPS Signals in Multipath and Jamming Environments," to appear in Signal Processing (Elsevier), 2008.

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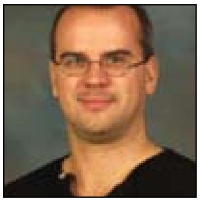

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